IGCSE · Cambridge (CIE) · Further Maths

Exam Questions

Functions

Language of Functions / Inverse Functions / Composite Functions

Medium (7 questions)	/34
Hard (5 questions)	/27
Very Hard (3 questions)	/26
Total Marks	**/87**

Medium Questions

1 (a)
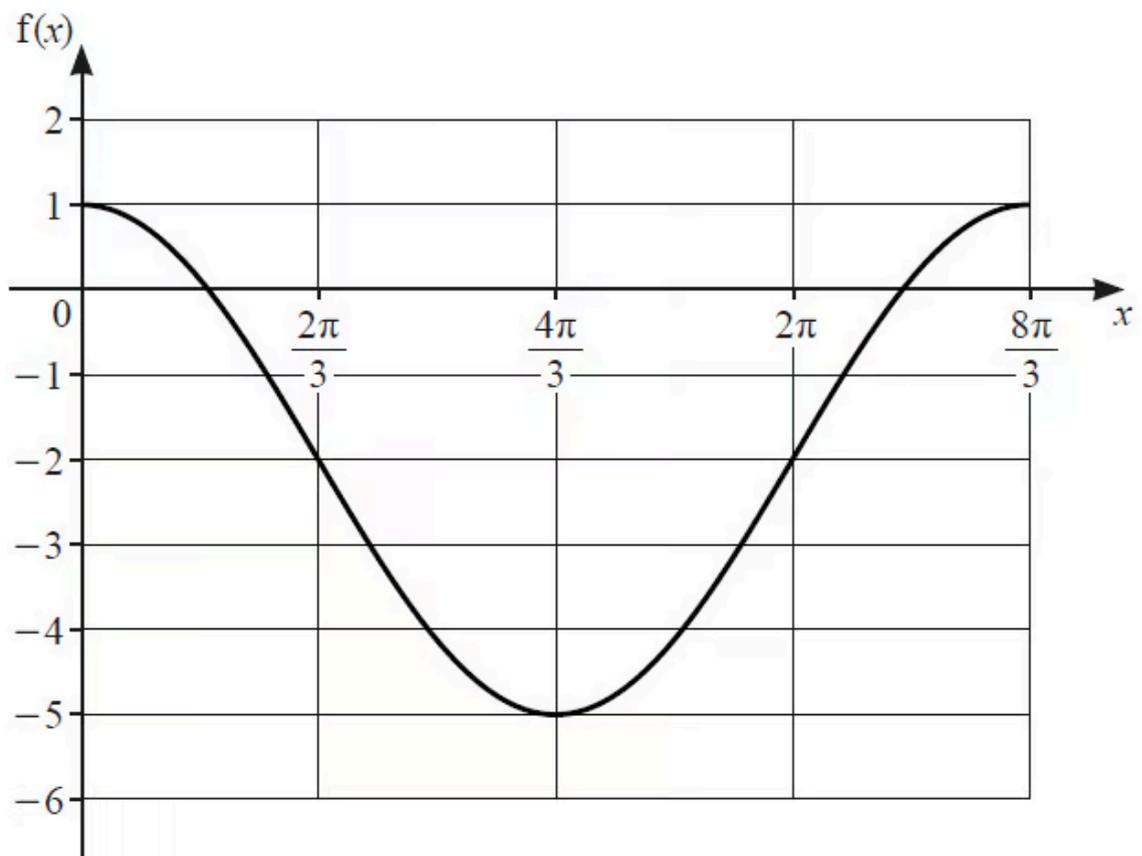
The diagram shows the graph of $f(x) = a \cos bx + c$ for $0 \le x \le \dfrac{8\pi}{3}$ radians. Explain why f is a function.

(1 mark)

(b) Write down the range of f.

(1 mark)

2 $g(x) = 3 + \dfrac{1}{x}$ for $x \ge 1$.

(i) Find an expression for $g^{-1}(x)$.

[2]

(ii) Write down the range of g^{-1}.

[1]

(iii) Find the domain of g^{-1}.

[2]

(5 marks)

3 (a) It is given that $f(x) = 5\ln(2x + 3)$ for $x > -\dfrac{3}{2}$.
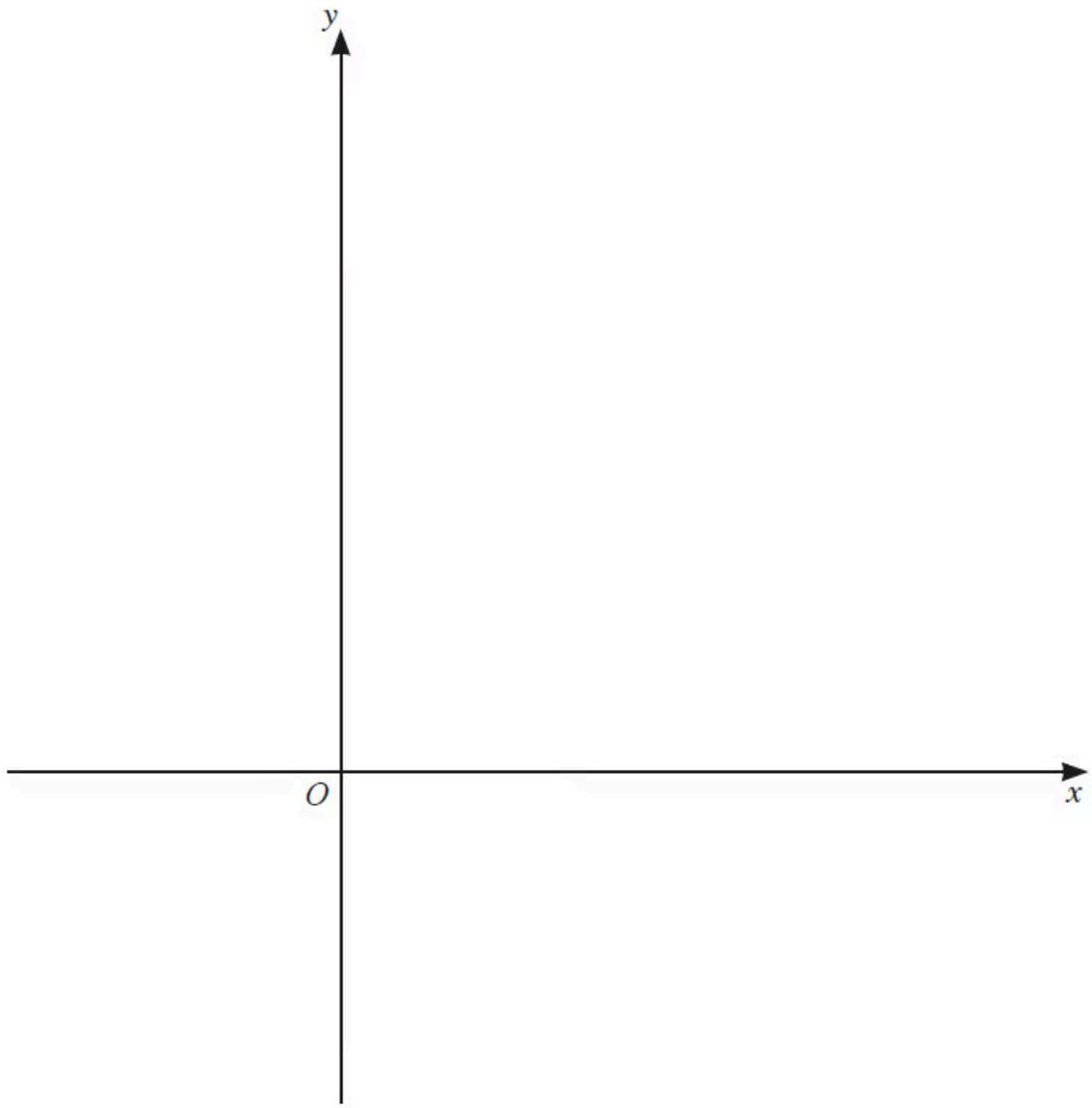
Write down the range of f.

(1 mark)

(b) Find f^{-1} and state its domain.

(3 marks)

(c) On the axes below, sketch the graph of $y = f(x)$ and the graph of $y = f^{-1}(x)$. Label

each curve and state the intercepts on the coordinate axes.

(5 marks)

4
$$f(x) = 4\ln(2x-1)$$

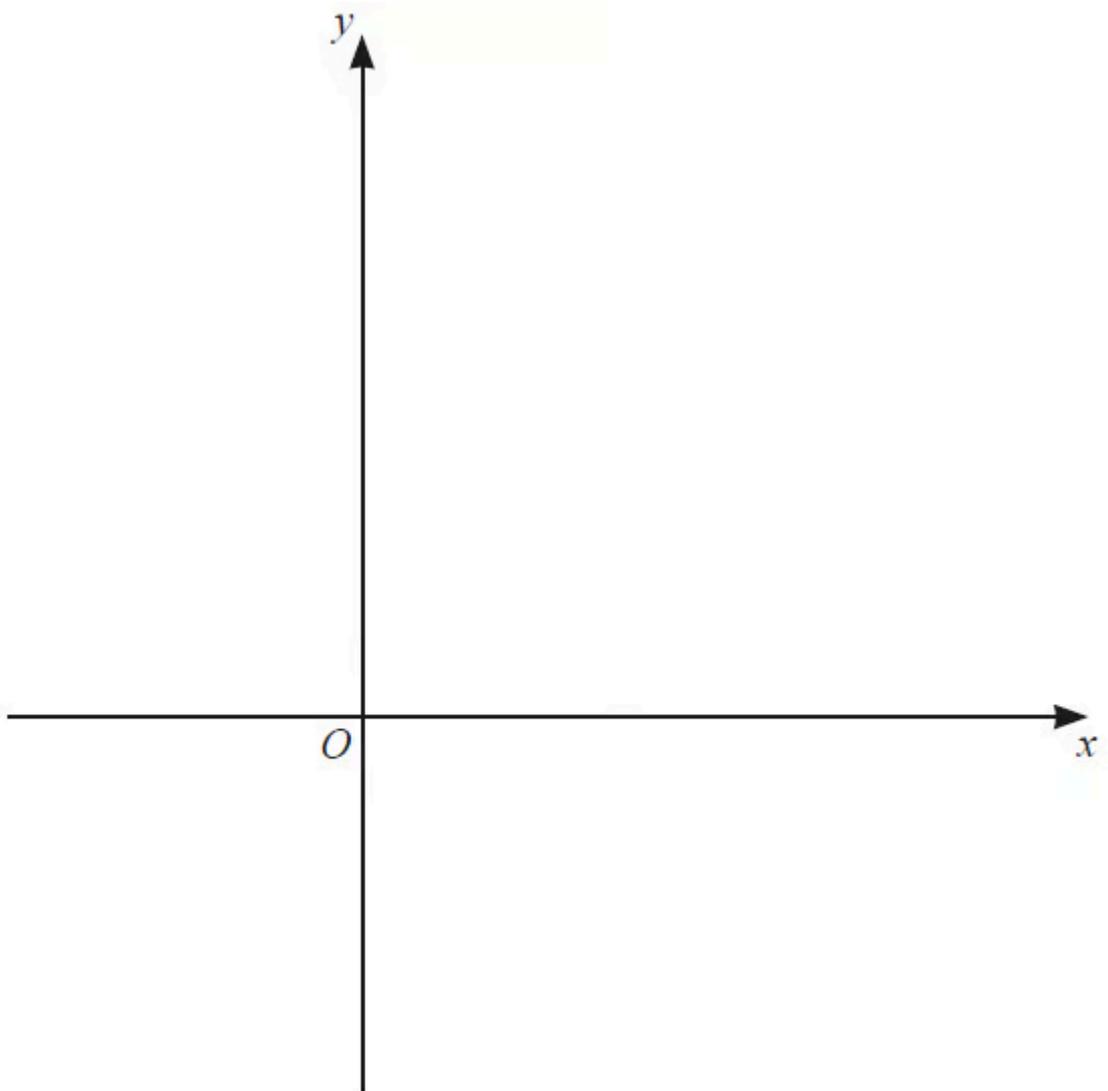(i) Write down the largest possible domain for the function f.

[1]

(ii) Find $f^{-1}(x)$ and its domain.

[3]

5 $h(x) = 2 \ln(3x - 1)$ for $x \geq \dfrac{2}{3}$.

The graph of $y = h(x)$ intersects the line $y = x$ at two distinct points. On the axes below, sketch the graph of $y = h(x)$ and hence sketch the graph of $y = h^{-1}(x)$.

(4 marks)

6 (a) Two functions are given by $f(x) = 1 + 8x$ where $x \in \mathbb{R}$ and $g(x) = 2 + \ln x$ where $x > 0$.

Find an expression for $f^{-1}(x)$.

(1 mark)

(b) State the geometric relationship between the graph of $y = f(x)$ and the graph of $y = f^{-1}(x)$.

(1 mark)

(c) Find $fg^2(1)$.

(2 marks)

(d) Explain why the function $gf(x)$ does not exist.

(1 mark)

7 (a) A curve has the equation $y = 5x^2 - 10x + 9$.

Write the equation in the form $y = p(x + q)^2 + r$ where p, q and r are constants to be found.

(2 marks)

(b) Hence find the coordinates of the turning point on the curve.

(1 mark)

(c) Find the range of the function $f(x) = 5x^2 - 10x + 9$ where $x \in \mathbb{R}$.

(1 mark)

(d) Explain why the inverse function $f^{-1}(x)$ does not exist.

(1 mark)

Hard Questions

1 It is given that $h(x) = a + \dfrac{b}{x^2}$, where a and b are non-zero constants.

(i) Explain why $-2 \leqslant x \leqslant 2$ is not a suitable domain for $h(x)$.

(ii) Given that $h(1) = 4$ and $h'(1) = 16$, find the values of a and b.

(3 marks)

2 (a)

$$f : x \mapsto (2x+3)^2 \text{ for } x > 0$$

Find the range of f.

(1 mark)

(b) Explain why f has an inverse.

(1 mark)

(c) Find f^{-1}.

(3 marks)

(d) State the domain of f^{-1}.

(1 mark)

(e) Given that $g : x \mapsto \ln(x+4)$ for $x > 0$, find the exact solution of $fg(x) = 49$.

(3 marks)

3
$$g(x) = x + 5 \text{ for } x \in \mathbb{R}$$
$$h(x) = \sqrt{2x - 3} \text{ for } x \geq \frac{3}{2}$$

Solve $gh(x) = 7$.

(3 marks)

4 (a) The functions f and g are defined as follows.

$$f(x) = x^2 + 4x \text{ for } x \in R$$
$$g(x) = 1 + e^{2x} \text{ for } x \in R$$

Find the range of f.

(2 marks)

(b) Write down the range of g.

(1 mark)

(c) Find the exact solution of the equation $fg(x) = 21$, giving your answer as a single logarithm

(4 marks)

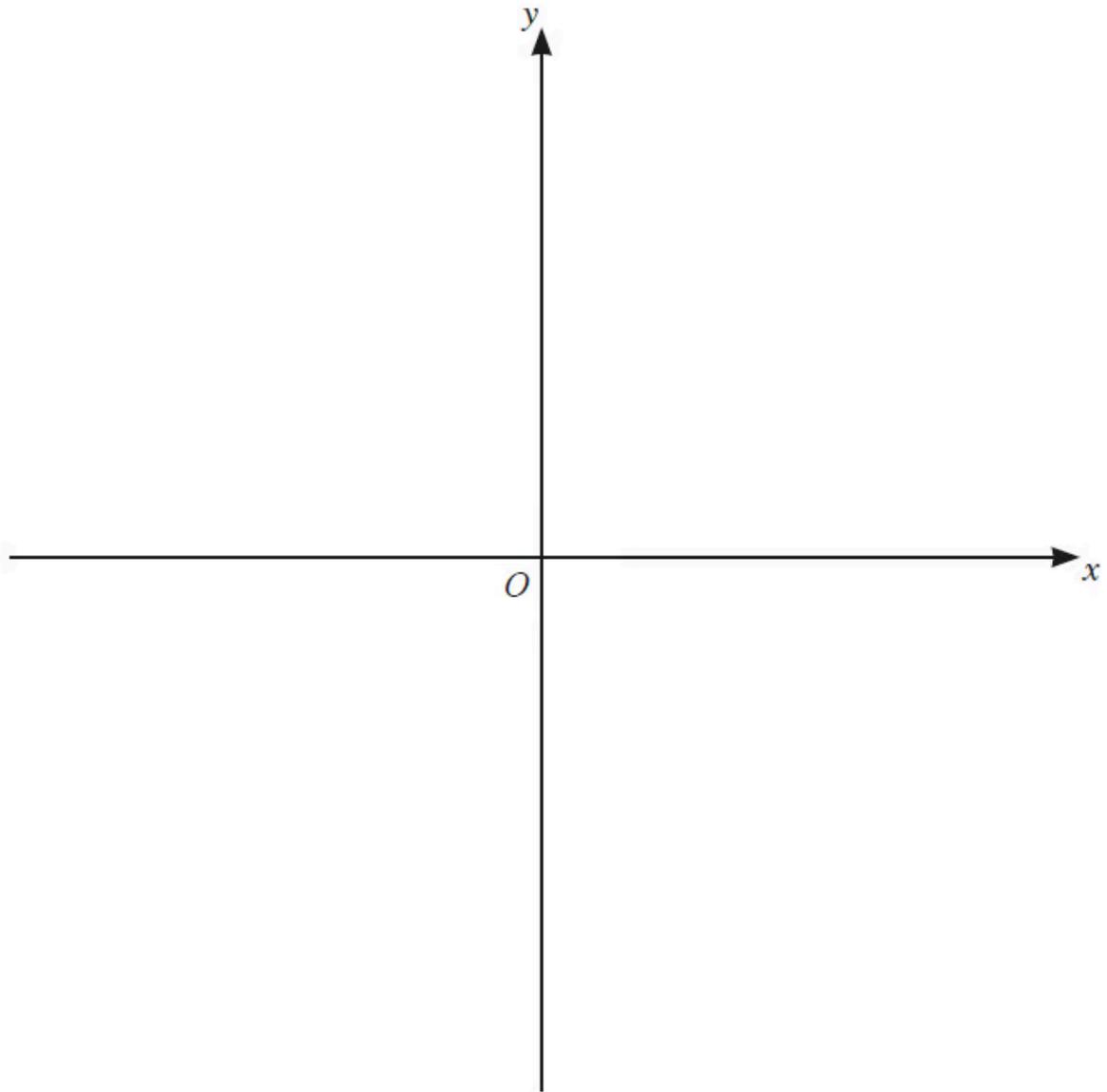**5 (a)** $f(x) = x^2 + 2x - 3$ for $x \geq -1$

Given that the minimum value of $x^2 + 2x - 3$ occurs when $x = -1$, explain why $f(x)$ has an inverse.

(1 mark)

(b) On the axes below, sketch the graph of $y = f(x)$ and the graph of $y = f^{-1}(x)$. Label each graph and state the intercepts on the coordinate axes.
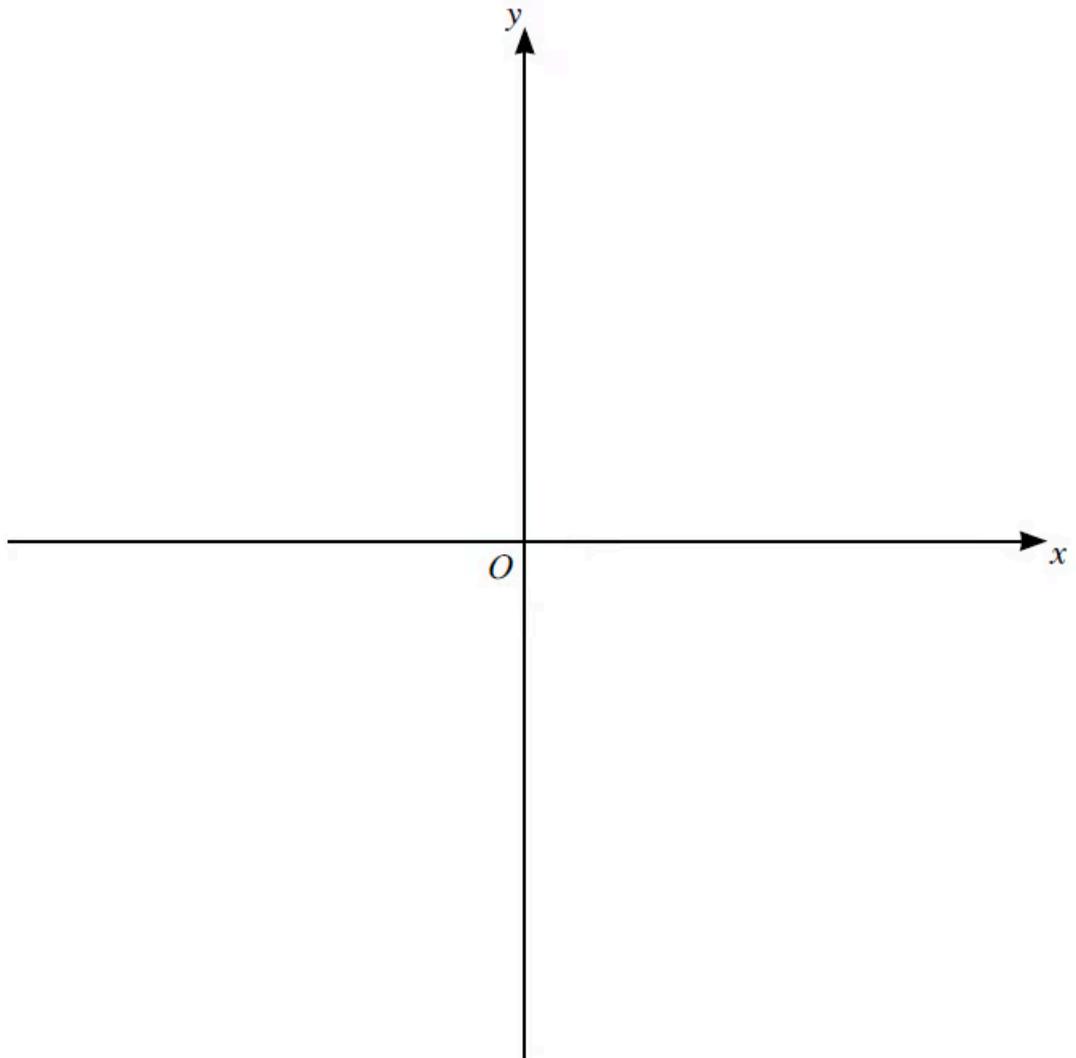

(4 marks)

Very Hard Questions

1 $f(x) = 3e^{2x} + 1$ for $x \in \mathbb{R}$

$g(x) = x + 1$ for $x \in \mathbb{R}$

(i) Write down the range of f and the range of g.

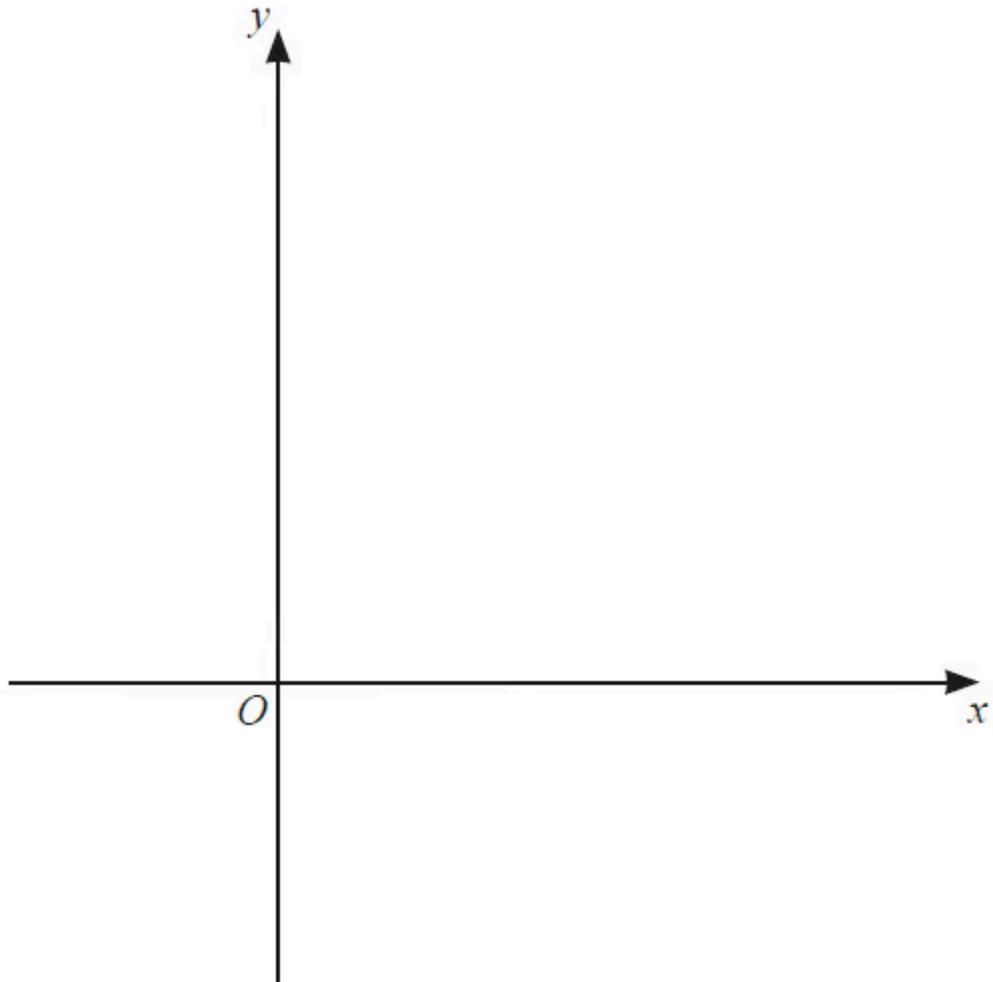(ii) Find $g^2(0)$.

(iii) Hence find $fg^2(0)$.

(iv) On the axes below, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$. State the intercepts with the coordinate axes and the equations of any asymptotes.

(9 marks)

2 (a) The function f is defined by $f(x) = \ln(2x+1)$ for $x \geq 0$.

Sketch the graph of $y = f(x)$ and hence sketch the graph of $y = f^{-1}(x)$ on the axes below.

(3 marks)

(b) The function g is defined by $g(x) = (x-4)^2 + 1$ for $x \leq 4$.

(i) Find an expression for $g^{-1}(x)$ and state its domain and range.

[4]

(ii) Find and simplify an expression for $fg(x)$.

[2]

(iii) Explain why the function gf does not exist.

(7 marks)

3 (a)

$$f(x) = 3 + e^x \text{ for } x \in \mathbb{R}$$
$$g(x) = 9x - 5 \text{ for } x \in \mathbb{R}$$

Find the range of f and of g.

(2 marks)

(b) Find the exact solution of $f^{-1}(x) = g'(x)$.

(3 marks)

(c) Find the solution of $g^2(x) = 112$.

(2 marks)