



IGCSE · Cambridge (CIE) · Further Maths

🕒 2 hours    ❓ 24 questions

Exam Questions

# Logarithmic & Exponential Functions

Exponential Functions / Logarithmic Functions / Laws of Logarithms / Exponential Equations / Transforming Relationships to Linear Form

Medium (7 questions)	/25
Hard (8 questions)	/44
Very Hard (9 questions)	/47
<b>Total Marks</b>	<b>/116</b>

# Medium Questions

1 Given that  $\log_a p + \log_a 5 - \log_a 4 = \log_a 20$ , find the value of  $p$ .

(2 marks)

2 In this question,  $a$ ,  $b$ ,  $c$  and  $d$  are positive constants.

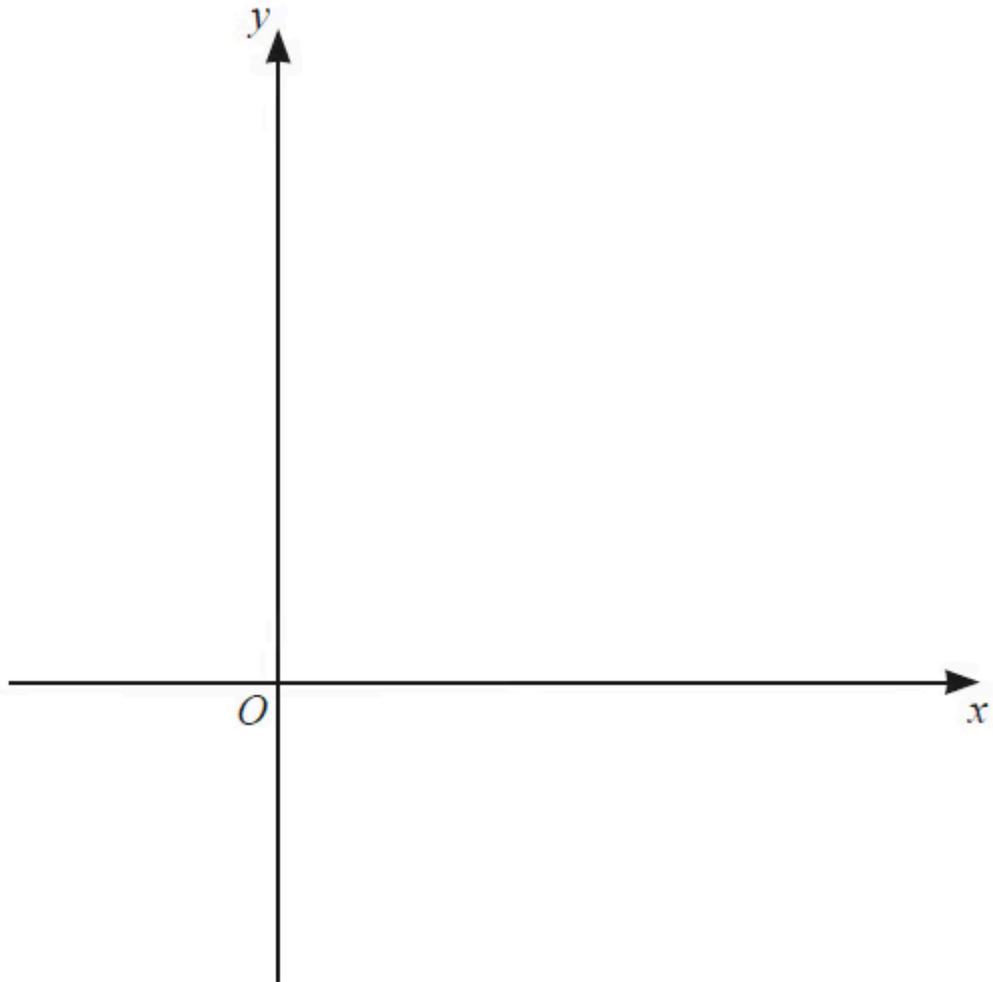
(i) It is given that  $y = \log_a(x+3) + \log_a(2x-1)$ . Explain why  $x$  must be greater than  $\frac{1}{2}$ .

(ii) Find the exact solution of the equation  $\frac{\log_a 6}{\log_a(y+3)} = 2$

(4 marks)

3 The function  $f$  is defined by  $f(x) = \ln(2x+1)$  for  $x \geq 0$ .

Sketch the graph of  $y = f(x)$  and hence sketch the graph of  $y = f^{-1}(x)$  on the axes below.



**(3 marks)**

4 Solve the equation  $\frac{9^{5x}}{27^{x-2}} = 243$

**(3 marks)**

5 Write  $3 \lg x + 2 - \lg y$  as a single logarithm.

(3 marks)

6 Using the substitution  $y = 2^x$ , or otherwise, solve  $2^{2x+1} - 2^{x+1} - 2^x + 1 = 0$ .

(4 marks)

7 (a) Solve the following equation.

$$\frac{1 + 3e^x}{e^x + e^{-x}} = 1$$

Give your answer(s) in the form  $p \ln q$  where  $p$  and  $q$  are integers.

(4 marks)

(b) Given that  $\log_2 y = c$ , express  $\log_2(2y) - \log_y 2$  in terms of  $c$  only.

(2 marks)

# Hard Questions

- 1 For variables  $x$  and  $y$ , plotting  $\ln y$  against  $\ln x$  gives a straight-line graph passing through the points  $(6, 5)$  and  $(8, 9)$ .

Show that  $y = e^p x^q$  where  $p$  and  $q$  are integers to be found.

(4 marks)

- 2 Variables  $x$  and  $y$  are such that, when  $\lg y$  is plotted against  $x^3$ , a straight line graph passing through the points  $(6,7)$  and  $(10,9)$  is obtained. Find  $y$  as a function of  $x$ .

(4 marks)

- 3 Given that  $\log_2 x + 2 \log_4 y = 8$ , find the value of  $xy$ .

(3 marks)

- 4 (a) Variables  $x$  and  $y$  are connected by the relationship  $y = Ax^n$ , where  $A$  and  $n$  are constants.

Transform the relationship  $y = Ax^n$  to straight line form.

**(2 marks)**

- (b)** When  $\ln y$  is plotted against  $\ln x$  a straight line graph passing through the points  $(0, 0.5)$  and  $(3.2, 1.7)$  is obtained.

Find the value of  $n$  and of  $A$ .

**(4 marks)**

- (c)** Find the value of  $y$  when  $x = 11$ .

**(2 marks)**

- 5 (a)** The population  $P$ , in millions, of a country is given by  $P = A \times b^t$ , where  $t$  is the number of years after January 2000 and  $A$  and  $b$  are constants. In January 2010 the population was 40 million and had increased to 45 million by January 2013.

Show that  $b = 1.04$  to 2 decimal places and find  $A$  to the nearest integer.

**(4 marks)**

- (b)** Find the population in January 2020, giving your answer to the nearest million.

**(1 mark)**

- (c)** In January of which year will the population be over 100 million for the first time?

**(3 marks)**

- 6 (a)** The number,  $b$ , of bacteria in a sample is given by  $b = P + Qe^{2t}$ , where  $P$  and  $Q$  are constants and  $t$  is time in weeks. Initially there are 500 bacteria which increase to 600 after 1 week.

Find the value of  $P$  and of  $Q$ .

**(4 marks)**

- (b)** Find the number of bacteria present after 2 weeks.

**(1 mark)**

- (c)** Find the first week in which the number of bacteria is greater than 1 000 000.

**(3 marks)**

**7**

$$f(x) = 3 + e^x \text{ for } x \in \mathbb{R}$$

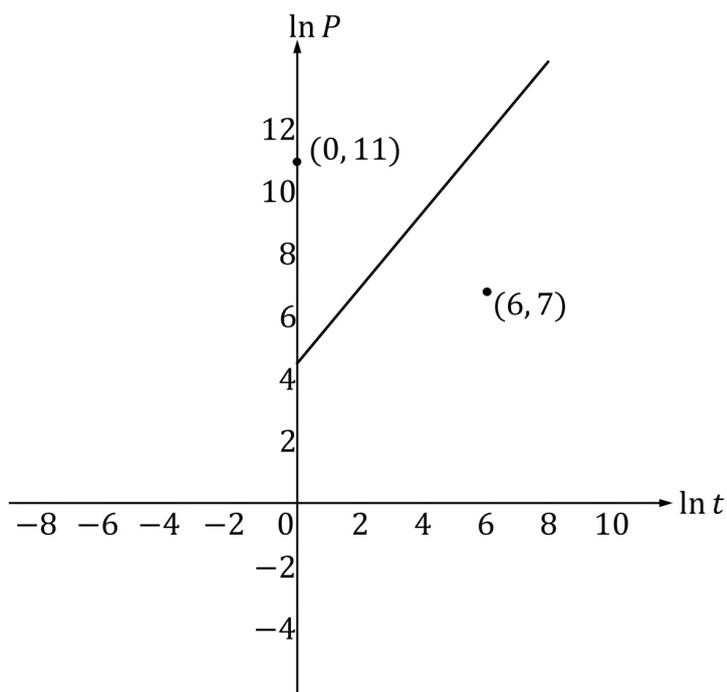
$$g(x) = 9x - 5 \text{ for } x \in \mathbb{R}$$

Find the exact solution of  $f^{-1}(x) = g'(x)$ .

**(3 marks)**

- 8** A mathematical model is used to predict the population of an island,  $P$ , over time,  $t$  years.

A plot of  $\ln P$  against  $\ln t$  gives a straight line that is the **perpendicular bisector** of the points  $(0, 11)$  and  $(6, 7)$ , as shown.



Find and simplify a formula for  $P$  in terms of  $t$ .

(6 marks)

# Very Hard Questions

1 Solve the equation  $3^{2x+1} + 8(3^x) - 3 = 0$ .

(3 marks)

2 Solve the equation  $4 \log_y 2 + \log_2 y = 4$ .

(3 marks)

3 Write the expression  $\log_a 9 + (\log_a b)(\log_{\sqrt{b}} 9a)$  in the form  $c + d \log_a 9$ , where  $c$  and  $d$  are integers.

(4 marks)

4 Find the exact solution of  $3^{2x} - 3^{x+1} - 4 = 0$ .

(4 marks)

5 Solve the simultaneous equations

$$10^{x+2y} = 5,$$

$$10^{3x+4y} = 50,$$

giving  $x$  and  $y$  in exact simplified form.

**(4 marks)**

6  $\log_a \sqrt{b} - \frac{1}{2} = \log_b a$ , where  $a > 0$  and  $b > 0$ .

Solve this equation for  $b$ , giving your answers in terms of  $a$ .

**(5 marks)**

7 Solve the simultaneous equations.

$$\log_3(x+y) = 2$$

$$2\log_3(x+1) = \log_3(y+2)$$

**(6 marks)**

- 8 (a)** It is known that  $y = A \times 10^{bx^2}$ , where  $A$  and  $b$  are constants. When  $\lg y$  is plotted against  $x^2$ , a straight line passing through the points (3.63, 5.25) and (4.83, 6.88) is obtained.

Find the value of  $A$  and of  $b$ .

**(4 marks)**

- (b)** Using your values of  $A$  and  $b$ , find the value of  $y$  when  $x = 2$ ,

**(2 marks)**

- (c)** Find the positive value of  $x$  when  $y = 4$ .

**(2 marks)**

9 (a)

$$\log_2(y+1) = 3 - 2 \log_2 x$$

$$\log_2(x+2) = 2 + \log_2 y$$

Show that  $x^3 + 6x^2 - 32 = 0$ .

(4 marks)

(b) Find the roots of  $x^3 + 6x^2 - 32 = 0$ .

(4 marks)

(c) Give a reason why only one root is a valid solution of the logarithmic equations. Find the value of  $y$  corresponding to this root.

(2 marks)