IGCSE · Cambridge (CIE) · Further Maths

36 mins · 7 questions

Exam Questions

Simultaneous Equations

Linear Simultaneous Equations / Quadratic Simultaneous Equations

Medium (2 questions)	/8
Hard (2 questions)	/12
Very Hard (3 questions)	/16
Total Marks	**/36**

Medium Questions

1 The line $y = 5x + 6$ meets the curve $xy = 8$ at the points A and B. Find the coordinates of A and of B.

Answer

Substituting the equation of the line into the equation of the curve and solving these equations simultaneously by substitution

$$x(5x + 6) = 8$$

Expanding the brackets and setting the equation equal to 0

$$5x^2 + 6x - 8 = 0$$

[1]

Factorising to find the solutions

$$(5x - 4)(x + 2) = 0$$

$$x = \frac{4}{5}, \; x = -2$$

Substituting to find the y- coordinates

$$y = 5\left(\frac{4}{5}\right) + 6 = 10$$

$$y = 5(-2) + 6 = -4$$

$\left(\frac{4}{5}, 10\right)$ [1]

$(-2, -4)$ [1]

(3 marks)

2 Find the coordinates of the points of intersection of the curves $x^2 = 5y - 1$ and $y = x^2 - 2x + 1$.

Answer

The curves intersect when they are equal

Make y the subject of the first curve

$$x^2 = 5y - 1$$

$$y = \frac{x^2 + 1}{5}$$

Now set the curves equal to each other

$$\frac{x^2 + 1}{5} = x^2 - 2x + 1$$

[1]

Multiply both sides by 5

$$x^2 + 1 = 5x^2 - 10x + 5$$

Rearrange to make the quadratic equation equal to 0

$$0 = 4x^2 - 10x + 4$$

[1]

Divide though by 2

$$2x^2 - 5x + 2 = 0$$

Solve the quadratic equation using the quadratic formula

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(2)}}{2(2)}$$

[1]

$$x = 2 \text{ or } x = \frac{1}{2}$$

Calculate the value of y at each of these points

$$\text{When } x = 2, \quad y = \frac{2^2 + 1}{5} = 1$$

$$\text{When } x = \frac{1}{2}, \quad y = \frac{\left(\frac{1}{2}\right)^2 + 1}{5} = \frac{1}{4}$$

$(2, 1)$ and $\left(\frac{1}{2}, \frac{1}{4}\right)$ [1]

(5 marks)

Hard Questions

1 (a) Solve the simultaneous equations

$$10^{x+2y} = 5,$$

$$10^{3x+4y} = 50,$$

giving x and y in exact simplified form.

Answer

Taking logs of both sides of each equation

$$\lg 10^{x+2y} = \lg 5$$

$$\lg 10^{3x+4y} = \lg 50$$

$\lg 10 = 1$, therefore

$$x + 2y = \lg 5$$

$$3x + 4y = \lg 50$$

[1]

Multiplying the first equation by 2 gives

$$2x + 4y = 2\lg 5$$

Subtracting this from the second equation gives

$$x = \lg 50 - 2\lg 5$$

[1]

Using the log laws that $a\log b = \log b^a$ and $\log m - \log n = \log \frac{m}{n}$

$$x = \lg 50 - \lg 25 = \lg 2$$

[1]

Substituting this into the equation $x + 2y = \lg 5$

$$\lg 2 + 2y = \lg 5$$

$$2y = \lg \frac{5}{2}$$

$x = \lg 2,\ y = \frac{1}{2}\lg\frac{5}{2}$ [1]

(4 marks)

(b) Solve $2x^{\frac{2}{3}} - x^{\frac{1}{3}} - 10 = 0$.

Answer

Let $y = x^{\frac{1}{3}}$, then

$$2y^2 - y - 10 = 0$$

$$(2y - 5)(y + 2) = 0$$

[1]

Solving the quadratic gives

$$y = \frac{5}{2} \text{ and } y = -2$$

Substituting $y = x^{\frac{1}{3}}$

$$x^{\frac{1}{3}} = \frac{5}{2} \text{ and } x^{\frac{1}{3}} = -2$$

[1]

Cubing both sides of each equation gives

$$x = \frac{125}{8} \text{ and } x = -8$$ [1]

(3 marks)

2 Find the coordinates of the points of intersection of the curve $x^2 + xy = 9$ and the line $y = \frac{2}{3}x - 2$.

Answer

To find the points of intersection, solve the equations simultaneously by substitution.

Substitute the equation of the line into the equation of the curve

$$x^2 + x\left(\frac{2}{3}x - 2\right) = 9$$

[1]

Expanding the brackets gives

$$x^2 + \frac{2}{3}x^2 - 2x = 9$$

Collecting like terms and setting the quadratic equal to 0

$$\frac{5}{3}x^2 - 2x - 9 = 0$$

Multiplying through by 3 gives

$$5x^2 - 6x - 27 = 0$$

[1]

To factorise, we look for a pair of numbers that has a product of −135 (from doing 5×-27) and a sum of −6. The pair of numbers would be −15 and 9 so we split the middle term

$$5x^2 - 15x + 9x - 27 = 0$$

Factorising each half of the equation

$$5x(x - 3) + 9(x - 3) = 0$$

We have a common factor of $x - 3$ and the remaining terms create the other factor

$$(5x + 9)(x - 3) = 0$$

[1]

Solving each bracket gives the x co-ordinates

$$x = -\frac{9}{5}, x = 3$$

Substituting these into either equation will give the y co-ordinates

$$y = \frac{2}{3}\left(-\frac{9}{5}\right) - 2 = -\frac{16}{5}$$

$$y = \frac{2}{3}(3) - 2 = 0$$

$\left(-\frac{9}{5}, -\frac{16}{5}\right)$ [1]

$(3, 0)$ [1]

(5 marks)

Very Hard Questions

1 Solve the simultaneous equations.

$$\log_3(x+y)=2$$

$$2\log_3(x+1)=\log_3(y+2)$$

Answer

The first equation is $\log_3(x+y)=2$

Use the inverse of $\log_3$ which is doing 3 to the power of both sides

$$x+y=3^2$$

$$(1) \qquad x+y=9$$

[1]

The second equation is $2\log_3(x+1)=\log_3(y+2)$

Move the 2 from in front to be a power

$$\log_3(x+1)^2=\log_3(y+2)$$

Use the inverse of $\log_3$ which is doing 3 to the power of both sides

$$(x+1)^2=y+2$$

[1]

Subtract 2 from both sides to make y the subject

$$(2) \qquad y=(x+1)^2-2$$

Solve equations (1) and (2) simultaneously by substituting equation (2) into equation (1)

$$x + (x + 1)^2 - 2 = 9$$

[1]

Expand the double bracket

$$x + x^2 + x + x + 1 - 2 = 9$$

Simplify the left hand side

$$x^2 + 3x - 1 = 9$$

Subtract 9 from both sides to make the quadratic equal 0

$$x^2 + 3x - 10 = 0$$

[1]

Solve by factorising (or using the quadratic formula)

$$(x + 5)(x - 2) = 0$$

$$x = -5 \text{ or } x = 2$$

[1]

Find y by substituting each value into equation (1)

When $x = -5$

$$y = 9 - (-5) = 14$$

Reject this as we cannot have $\log_3(\text{negative})$

When $x = 2$

$$y = 9 - 2 = 7$$

$x = 2$ and $y = 7$ [1]

(6 marks)

2 Solve the simultaneous equations.

$$x^2 + 3xy = 4$$

$$2x + 5y = 4$$

Answer

Solve simultaneously by using a substitution method.

For example, by rearranging the second equation to make y the subject and substituting into the first equation.

$$2x + 5y = 4$$

$$5y = 4 - 2x$$

$$y = \frac{4 - 2x}{5}$$

Substitute into the first equation.

$$x^2 + 3x\left(\frac{4 - 2x}{5}\right) = 4$$

[1]

Expand the brackets.

$$x^2 + \frac{12x - 6x^2}{5} = 4$$

Multiply through by 5.

$$5x^2 + 12x - 6x^2 = 20$$

Rearrange.

$$x^2 - 12x + 20 = 0$$

[1]

Solve by factorising.

$$(x-10)(x-2)=0$$

[1]

$$x=10, \quad x=2$$

[1]

Substitute each back into the linear equation to find the corresponding y values.

$$\begin{aligned} 2x+5y&=4 \\ 2(10)+5y&=4 \\ 5y&=4-20 \\ y&=-\frac{16}{5} \end{aligned}$$

$$\begin{aligned} 2x+5y&=4 \\ 2(2)+5y&=4 \\ 5y&=4-4 \\ y&=0 \end{aligned}$$

$$y=-\frac{16}{5}, \quad y=0$$

$\mathbf{x=2, \quad y=0 \ \text{and} \ x=10, \ y=-\frac{16}{5}}$ [1]

(5 marks)

3 Solve the following simultaneous equations.

$$3^x \times 9^{y-1} = 243$$

$$8 \times 2^{y-\frac{1}{2}} = \frac{2^{2x+1}}{4\sqrt{2}}$$

Answer

Rewrite the first equation in powers of 3.

$$3^x \times (3^2)^{y-1} = 3^5$$
$$3^x \times 3^{2y-2} = 3^5$$
$$3^{x+2y-2} = 3^5$$

[1]

Equate the powers.

$$x + 2y - 2 = 5$$

$$x + 2y = 7$$

[1]

Express the second equation in powers of 2.

$$(2^3) \times 2^{y-\frac{1}{2}} = \frac{2^{2x+1}}{(2^2)\left(2^{\frac{1}{2}}\right)}$$

$$2^{y+\frac{5}{2}} = \frac{2^{2x+1}}{2^{\frac{5}{2}}}$$

$$2^{y+\frac{5}{2}} = 2^{2x-\frac{3}{2}}$$

Equate the powers.

$$y+\frac{5}{2}=2x-\frac{3}{2}$$

$$2x-y=4$$

[1]

Solve the two equations simultaneously. For example, by rearranging the first equation and using a substitution method.

$$x+2y=7$$
$$x=7-2y$$

Substitute into the second equation.

$$2(7-2y)-y=4$$
$$14-4y-y=4$$
$$-5y=4-14$$
$$y=\frac{10}{5}$$
$$y=2$$

[1]

Substitute into one of the equations to solve for x.

$$x=7-2(2)$$
$$x=3$$

$x=3,\ \ y=2$ [1]

(5 marks)