



IGCSE · Cambridge (CIE) · Further Maths

🕒 2 hours    ❓ 11 questions

Exam Questions

# Arithmetic & Geometric Progressions

Language of Sequences & Series / Arithmetic Progressions / Geometric Progressions

Medium (2 questions)	/17
Hard (7 questions)	/73
Very Hard (2 questions)	/22
<b>Total Marks</b>	<b>/112</b>

# Medium Questions

**1 (a)** The 7th and 10th terms of an arithmetic progression are 158 and 149 respectively.

Find the common difference and the first term of the progression.

**(3 marks)**

**(b)** Find the least number of terms of the progression for their sum to be negative.

**(3 marks)**

**2 (a)** The sum of the first 4 terms of an arithmetic progression is 38 and the sum of the next 4 terms is 86. Find the first term and the common difference.

**(5 marks)**

**(b)** The third term of a geometric progression is 12 and the sixth term is -96. Find the sum of the first 10 terms of this progression.

**(6 marks)**

# Hard Questions

- 1 (a)** In an arithmetic progression, the 5th term is equal to  $\frac{1}{3}$  of the 16th term. The sum of the 5th term and the 16th term is equal to 33.

Find the sum of the first 10 terms of this progression.

**(6 marks)**

- (b)** In a geometric progression, the sum of the first two terms is equal to 16. The sum to infinity is equal to 25.

Find the possible values of the first term.

**(6 marks)**

**2 (a)** The sum of the first two terms of a geometric progression is 10 and the third term is 9.

(i) Find the possible values of the common ratio and the first term.

[5]

(ii) Find the sum to infinity of the convergent progression.

[1]

**(6 marks)**

**(b)** In an arithmetic progression,  $u_1 = -10$  and  $u_4 = 14$ . Find

$u_{100} + u_{101} + u_{102} + \dots + u_{200}$  the sum of the 100th to the 200th terms of the progression.

**(4 marks)**

**3 (a)** An arithmetic progression has a second term of -14 and a sum to 21 terms of 84. Find the first term and the 21st term of this progression.

**(5 marks)**

**(b)** A geometric progression has a second term of  $27p^2$  and a fifth term of  $p^5$ . The common ratio,  $r$ , is such that  $0 < r < 1$ .

(i) Find  $r$  in terms of  $p$ .

[2]

(ii) Hence find, in terms of  $p$ , the sum to infinity of the progression.

[3]

(iii) Given that the sum to infinity is 81, find the value of  $p$ .

[2]

**(7 marks)**

**4 (a)** The first 5 terms of a sequence are given below.

4 -2 1 -0.5 0.25

(i) Find the 20th term of the sequence.

[2]

(ii) Explain why the sum to infinity exists for this sequence and find the value of this sum.

[2]

**(4 marks)**

**(b)** The tenth term of an arithmetic progression is 15 times the second term. The sum of the first 6 terms of the progression is 87.

(i) Find the common difference of the progression.

[4]

(ii) For this progression, the  $n$ th term is 6990. Find the value of  $n$ .

[3]

(7 marks)

**5 (a)** An arithmetic progression has a first term of 7 and a common difference of 0.4. Find the least number of terms so that the sum of the progression is greater than 300.

**(4 marks)**

**(b)** The sum of the first two terms of a geometric progression is 9 and its sum to infinity is 36. Given that the terms of the progression are positive, find the common ratio.

**(4 marks)**

- 6 (a)** An arithmetic progression has a second term of 8 and a fourth term of 18. Find the least number of terms for which the sum of this progression is greater than 1560.

**(6 marks)**

- (b)** A geometric progression has a sum to infinity of 72. The sum of the first 3 terms of this progression is  $\frac{333}{8}$ .

(i) Find the value of the common ratio.

[5]

(ii) Hence find the value of the first term.

[1]

**(6 marks)**

**7 (a)** A geometric progression has a first term of 3 and a second term of 2.4. For this progression, find the sum of the first 8 terms.

**(3 marks)**

**(b)** Find the sum to infinity.

**(1 mark)**

**(c)** Find the least number of terms for which the sum is greater than 95% of the sum to infinity.

**(4 marks)**

# Very Hard Questions

**1 (a)** The 2nd, 8th and 44th terms of an arithmetic progression form the first three terms of a geometric progression. In the arithmetic progression, the first term is 1 and the common difference is positive.

(i) Show that the common difference of the arithmetic progression is 5.

(ii) Find the sum of the first 20 terms of the arithmetic progression.

**(7 marks)**

**(b)** (i) Find the 5th term of the geometric progression.

(ii) Explain whether or not the sum to infinity of this geometric progression exists.

**(3 marks)**

**2 (a)** In an arithmetic sequence, the 20<sup>th</sup> term is 10% of the sum of the first 25 terms.

The sum of the 20<sup>th</sup> term and the 21<sup>st</sup> term is 73.

Find the 8<sup>th</sup> term.

**(6 marks)**

**(b)** In a geometric sequence, the sum to infinity is 64 and the sum of the first 7 terms is  $\frac{127}{2}$ .

Find the ratio of the 4<sup>th</sup> term to the 7<sup>th</sup> term, giving your answer in the form  $m : 1$ .

**(6 marks)**