IGCSE · Cambridge (CIE) · Further Maths

1 hour 14 questions

Exam Questions

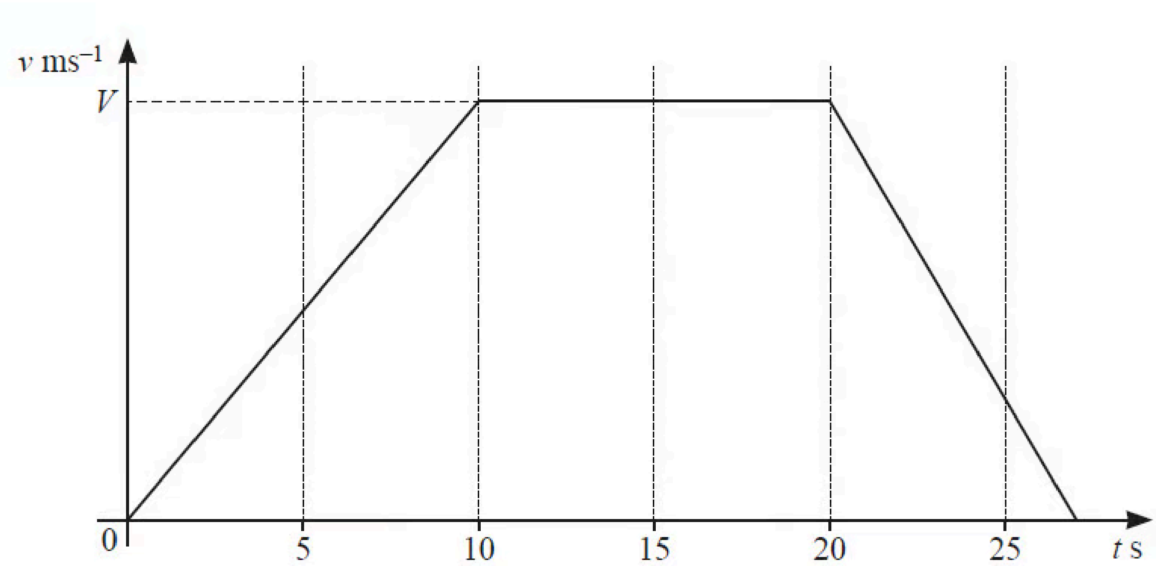
Calculus for Kinematics

Kinematics Toolkit / Calculus for Kinematics / Sketching Travel Graphs

Medium (5 questions)	/26
Hard (8 questions)	/49
Very Hard (1 question)	/8
Total Marks	**/83**

Medium Questions

1



The diagram shows the velocity–time graph for a particle Q travelling in a straight line with velocity $v\ \mathrm{ms}^{-1}$ at time t s. The particle accelerates at $3.5\ \mathrm{ms}^{-2}$ for the first 10 s of its motion and then travels at constant velocity, $V\ \mathrm{ms}^{-1}$, for 10 s. The particle then decelerates at a constant rate and comes to rest. The distance travelled during the interval $20 \le t \le 25$ *is* 112.5 m.

(i) Find the value of V.

[1]

(ii) Find the velocity of Q when $t = 25$.

[3]

(iii) Find the value of t when Q comes to rest.

[3]

Answer

i) Use the first section of the graph when the particle is accelerating

The acceleration is the gradient of the graph

$$\text{acceleration} = \frac{\text{final speed} - \text{initial speed}}{\text{time}}$$

$$3.5 = \frac{V - 0}{10}$$

$$V = 35 \text{ ms}^{-1}$$

35 [1]

ii) Use the area of the trapezium to find the unknown velocity

The area of the trapezium = distance travelled = 112.5 m

$$\text{Area of trapezium} = \frac{1}{2}(a + b)h$$

$$112.5 = \frac{1}{2}(35 + b)5$$

$$225 = (35 + b)5$$

$$45 = 35 + b$$

$$b = 10 \text{ ms}^{-1}$$

10 [1]

iii) Find the deceleration of the particle

Initial velocity = 35

Final velocity = 10

Time = 5

$$\text{deceleration} = \frac{\text{final speed} - \text{initial speed}}{\text{time}}$$

$$\text{deceleration} = \frac{35 - 10}{5} = 5\text{ms}^{-2}$$

[1]

Use the gradient of the last section of the graph

$$5 = \frac{35}{t - 20}$$

[1]

$$t - 20 = \frac{35}{5}$$

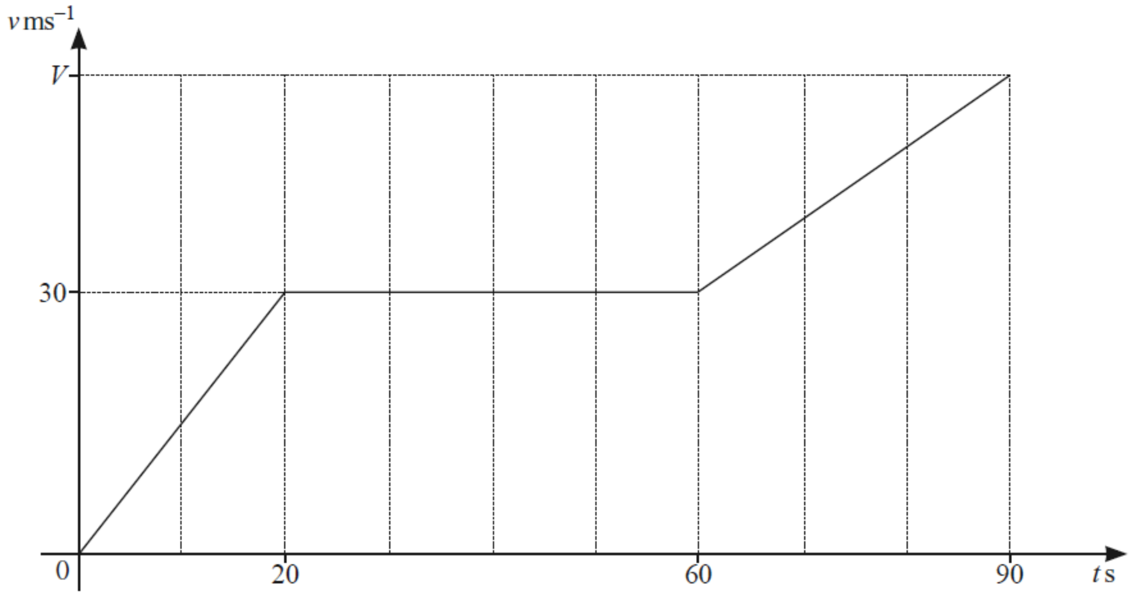
$$t - 20 = 7$$

$$t = 27\text{s}$$

27 [1]

(7 marks)

2



The diagram shows the velocity–time graph of a particle P that travels 2775 m in 90 s, reaching a final velocity of $V\,\text{ms}^{-1}$.

(i) Find the value of V.

[3]

(ii) Write down the acceleration of P when $t = 40$.

[1]

Answer

(i) The area under the line on a velocity–time graph is equal to the displacement of the object.

Since we are given the displacement, 2775m, we can find the final velocity, *V*, by finding the area under the graph. equating it to 2775 and then solving to find *V*.

Split the area under the graph into two trapeziums, find their areas, add them together and equate to 2775.

$$\frac{1}{2}(60+40)(30)+\frac{1}{2}(30+V)(30)=2775$$

for an attempt to find the area under the graph [1]

for equating to 2775 [1]

Rearrange and solve for *V*.

$$\begin{aligned}1500+15(30+V)&=2775\\15(30+V)&=1275\\30+V&=85\\V&=55\end{aligned}$$

$V = 55\ \text{ms}^{-1}$ [1]

(ii) Examine the graph.

When $t = 40$, the graph is a horizontal line with a gradient of 0. Therefore, the acceleration at this time is 0.

0 [1]

(4 marks)

3 The displacement, x m, of a particle from a fixed point at time t s is given by

$x = 6\cos\left(3t + \frac{\pi}{3}\right)$.

Find the acceleration of the particle when $t = \frac{2\pi}{3}$.

Answer

We need to differentiate $6\cos\left(3t + \frac{\pi}{3}\right)$ twice.

Differentiate once to find velocity, v.

$$\frac{dx}{dt} = -18\sin\left(3t + \frac{\pi}{3}\right)$$

[1]

Differentiate again to find acceleration, a.

$$\frac{d^2x}{dt^2} = -54\cos\left(3t + \frac{\pi}{3}\right)$$

[1]

Substitute $t = \frac{2\pi}{3}$ into the equation.

$$a = -54\cos\left(3\left(\frac{2\pi}{3}\right) + \frac{\pi}{3}\right)$$

$$a = -54\cos\left(\frac{7\pi}{3}\right)$$

$a = -27$ [1]
(3 marks)

4 (a) A particle P moves in a straight line such that, t seconds after passing through a fixed point O, its acceleration, $a\ \text{ms}^{-2}$, is given by $a = -6$. When $t = 0$, the velocity of P is $18\ \text{ms}^{-1}$.

Find the time at which P comes to instantaneous rest.

Answer

The particle moves in a straight line so we have a linear relationship between the initial and final velocity, which is dependent upon the acceleration

Form a linear equation

$$v = -6t + c$$

[1]

The constant is the initial velocity which is 18

$$v = -6t + 18$$

[1]

Substitute the final velocity, because we know the particle is at rest

$$0 = -6t + 18$$

$$0 = (-6)t + 18$$

Solve to find t

$$t = \frac{18}{6} = 3$$

$t = 3$ [1]
(3 marks)

(b) Find the distance travelled by P in the 3rd second.

Answer

Displacement (s) is the integral of velocity. From part (a), $v = 18 - 6t$.
The third second is between $t = 2$ and $t = 3$.

$$s = \int_2^3 (18 - 6t)\,\mathrm{d}t = [18t - 3t^2]_2^3$$

[1]

Apply the limits.

$$s = [18 \times 3 - 3 \times 3^2] - [18 \times 2 - 3 \times 2^2]$$
$$s = 27 - 24$$

[1]

As positive, distance will be the same as displacement.

***P* travels a distance of 3 metres in the third second** [1]

(3 marks)

5 The acceleration, $a\ \mathrm{ms}^{-2}$, of a particle Q travelling in a straight line, is given by $a = 6\cos 2t$ at time t s. When $t = 0$ the particle is at point O and is travelling with a velocity of $10\ \mathrm{ms}^{-1}$.

(i) Find the velocity of Q at time t.

[3]

(ii) Find the displacement of Q from O at time t.

[3]

Answer

(i) To find the expression for velocity, *v*, integrate the expression for acceleration.

$$\int (6\cos 2t)\,\mathrm{d}t = 3\sin 2t + C$$

$$v = 3\sin 2t + C$$

[1]

To find C, substitute $t = 0$ and $v = 10$ into the equation.

$$10 = 3\sin(2 \times 0) + C$$

Simplify, then rearrange, to find C.

$$C = 10$$

[1]

Substitute $C = 10$ into the equation for velocity.

$v = 3\sin 2t + 10$ [1]

ii) To find the expression for displacement, s, integrate the expression for velocity.

$$\int (3\sin 2t + 10)\,\mathrm{d}t$$

Break up the integral.

$$\int (3\sin 2t)\,\mathrm{d}t + \int 10\,\mathrm{d}t$$

Integrate $3\sin 2t$ with respect to t.

$$\int (3\sin 2t)\,\mathrm{d}t = \frac{-3\cos 2t}{2} + d$$

$$= -\frac{3}{2}(\cos 2t) + d$$

Combine with the integral of 10.

$$s = -\frac{3}{2}(\cos 2t) + 10t + d$$

[1]

To find d, substitute $s = 0$ and $t = 0$ into the equation.

$$0 = -\frac{3}{2}(\cos 0) + 10(0) + d$$
$$d = \frac{3}{2}$$

[1]

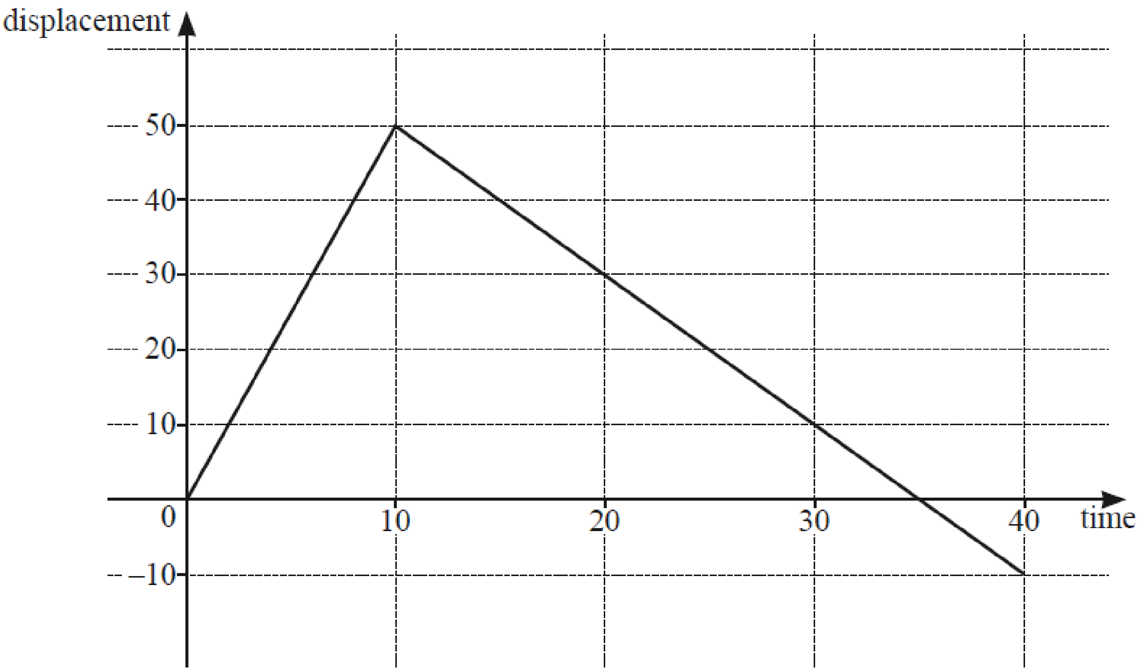
Substitute $d = \frac{3}{2}$ into the equation for displacement.

$$s = -\frac{3}{2}(\cos 2t) + 10t + \frac{3}{2} \text{ [1]}$$

(6 marks)

Hard Questions

1 In this question, all lengths are in metres and time, t, is in seconds.



The diagram shows the displacement–time graph for a runner, for $0 \leqslant t \leqslant 40$.

(i) Find the distance the runner has travelled when $t = 40$.

(ii) On the axes, draw the corresponding velocity–time graph for the runner, for $0 \leqslant t \leqslant 40$.

Answer

i) $0 - 10$ seconds, the runner travels 50m.
$10 - 40$ seconds, the runner travels 60m.

$$50 + 60$$

110 m [1]

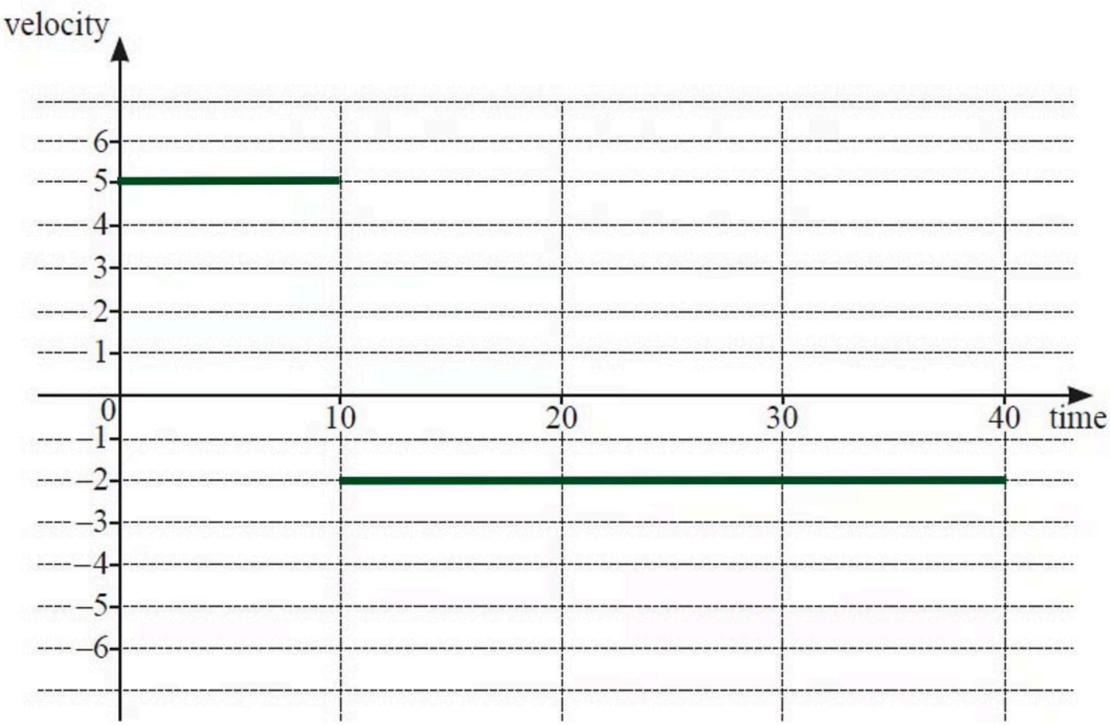
ii) Work out the gradient for each line to find velocity.
For $0 - 10$ seconds

$$\frac{50}{10} = 5 \text{ m/s}$$

For $10 - 40$ seconds

$$\frac{-60}{30} = -2 \text{ m/s}$$

Draw on the axes provided.



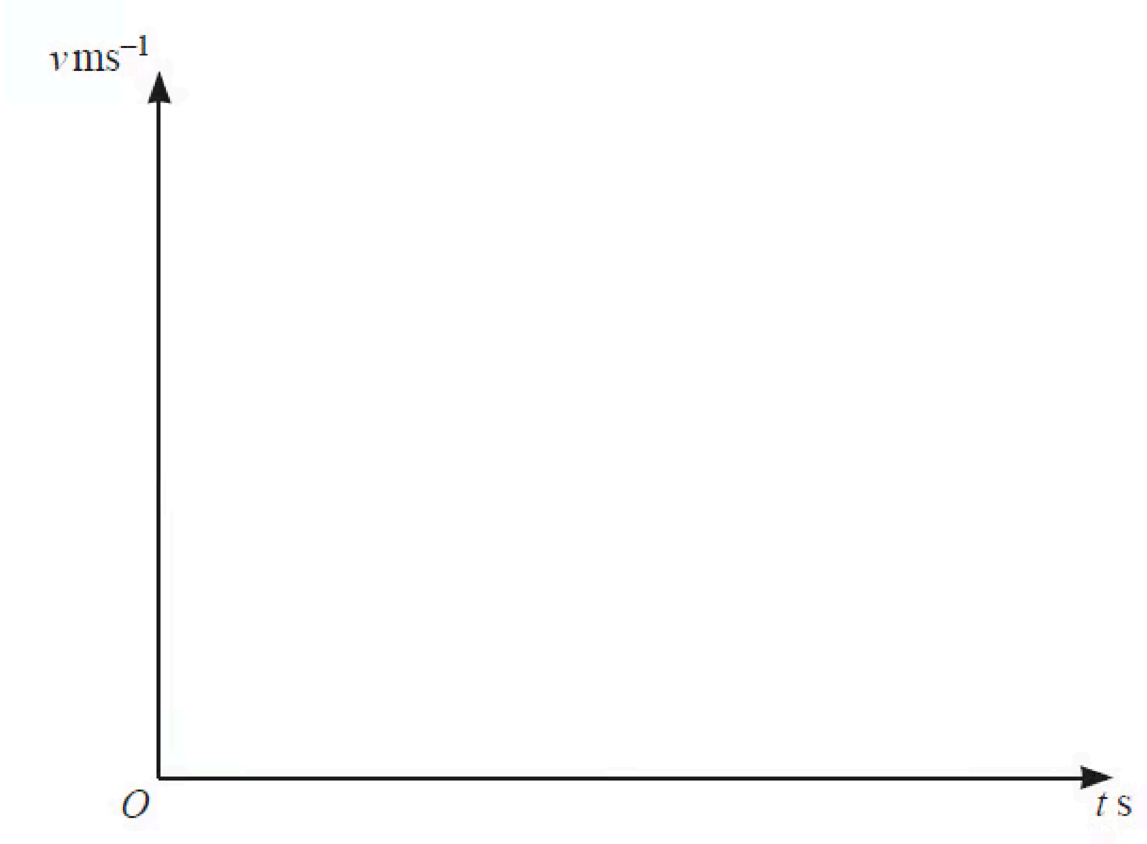
for a line joining $(0, 5)$ *and* $(10, 5)$ [1]

for a line joining $(10, -2)$ *and* $(40, -2)$ [1]

(3 marks)

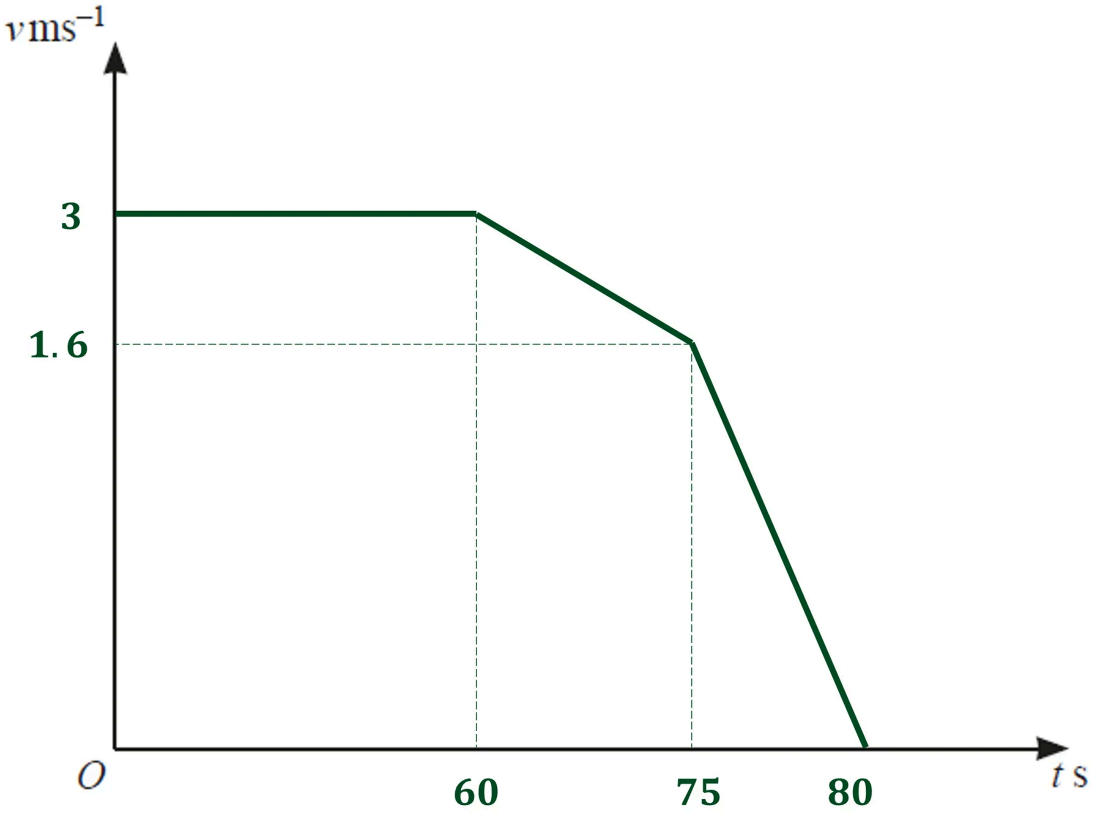
2 (a) A particle travels in a straight line. As it passes through a fixed point O, the particle is travelling at a velocity of 3 ms^{-1}. The particle continues at this velocity for 60 seconds then decelerates at a constant rate for 15 seconds to a velocity of 1.6 ms^{-1}. The particle then decelerates again at a constant rate for 5 seconds to reach point A , where it stops.

Sketch the velocity-time graph for this journey on the axes below.



Answer

If the graph is travelling at a constant velocity, this would be a horizontal line on the graph. A deceleration would have a negative gradient. The final section of the graph should have a steeper negative gradient as the particle has stopped after 5 seconds.



correct shape with three distinct sections [1]

3 and 1.6 on vertical axis [1]

60, 75 and 80 on horizontal axis [1]
(3 marks)

(b) Find the distance between O and A.

Answer

The distance is the area under the graph.
Finding the area of the first section of the graph, the rectangle

$$60 \times 3 = 180$$

Finding the area of the second section of the graph, the trapezium

$$\frac{1}{2}(3 + 1.6) \times 15 = 34.5$$

Finding the area of the third section of the graph, the triangle

$$\frac{1}{2} \times 1.6 \times 5 = 4$$

attempting at least two areas [1]

all areas calculated correctly [1]

Finding the sum of these three areas

218.5 m [1]

(3 marks)

(c) Find the deceleration in the last 5 seconds.

Answer

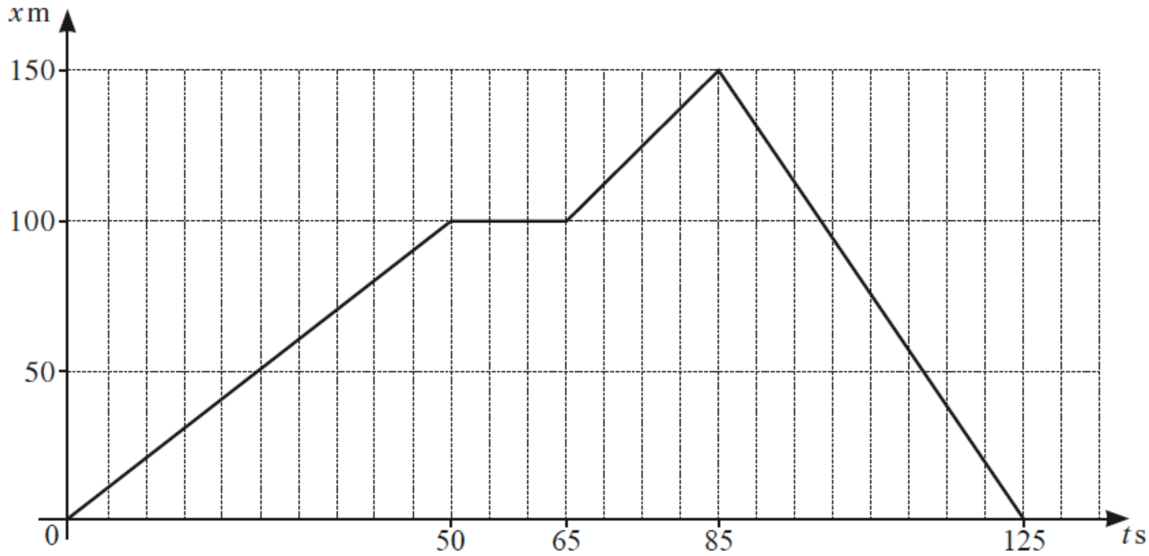
To find the deceleration, calculate the gradient of the line

$$\frac{1.6}{5}$$

$\mathbf{0.32\ ms^{-2}}$ [1]

(1 mark)

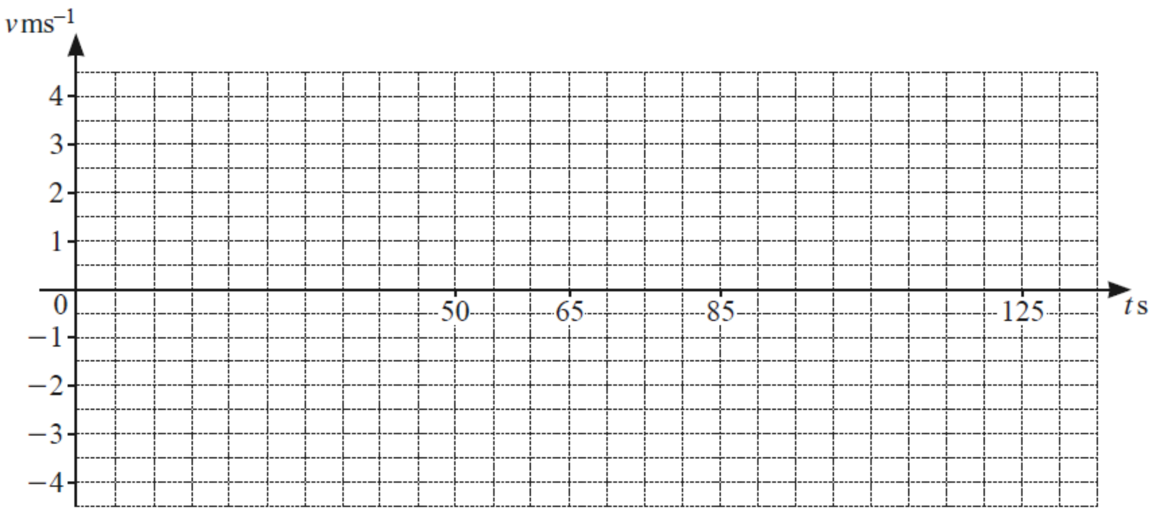
3



The diagram shows the $x-t$ graph for a runner, where displacement, x, is measured in metres and time, t, is measured in seconds.

(i) On the axes below, draw the $v-t$ graph for the runner.

[3]



(ii) Find the total distance covered by the runner in 125 s.

[1]

Answer

(i) To find velocity, v, divide displacement, x, by time, t.

For $0 \le t \le 50$

$$100 \div 50 = 2$$

For $50 \le t \le 65$

$$0 \div 15 = 0$$

For $65 \le t \le 85$

$$50 \div 20 = 2.5$$

For $85 \le t \le 125$

$$-150 \div 40 = -3.75$$

Draw the lines on the given axes.

for correct line at $v = 2$ [1]

for correct line at $v = 2.5$ [1]

for correct line at $v = 3.75$ *and* $v = 0$ [1]

(ii) $0 \le t \le 50$, the distance covered is 100m.

$50 \le t \le 65$, the distance covered is 0m.

$65 \le t \le 85$, the distance covered is 50m.

$85 \le t \le 125$, the distance covered is 150m.

$$100 + 0 + 50 + 150 = 300$$

300m [1]

(4 marks)

4 A particle P moves in a straight line such that its displacement, x m, from a fixed point O at time t s is given by $x = 10\sin 2t - 5$.

(i) Find the speed of P when $t = \pi$.

[1]

(ii) Find the value of t for which P is first at rest.

[2]

(iii) Find the acceleration of P when it is first at rest.

[2]

Answer

i) The differential of displacement is velocity

$$(v =)\frac{dx}{dt} = 20\cos 2t$$

At time $t = \pi$

$$v = 20\cos(2\pi) = 20 \times 1 = 20$$

As this is positive, speed will be the same

Speed of particle *P* at time π seconds is $20\ \mathrm{m\ s^{-1}}$ [1]

ii) When P is at rest, $v = 0$

Use the equation from part (i)

$$20\cos 2t = 0$$

[1]

$$\cos 2t = 0$$

$$2t = \cos^{-1}(0) = \frac{\pi}{2}$$

Divide by 2 to find t

$$t = \frac{\pi}{4}$$

$\frac{\pi}{4}$ s [1]

iii) To find the acceleration, differentiate the velocity

$$v = 20\cos 2t$$

$$\frac{dv}{dt} = -40\sin 2t$$

[1]

Calculate the acceleration when $t = \frac{\pi}{4}$

$$\frac{dv}{dt} = -40\sin\left(2 \times \frac{\pi}{4}\right) = -40$$

-40 ms^{-2} [1]

(5 marks)

5 (a) In this question, the units are metres and seconds. A particle P is travelling in a straight line.

Its acceleration, a, away from a fixed point O, at time t, is given by $a = (3t + 2)^{-\frac{1}{3}}$, where $t \geqslant 0$. When $t = 2$, P is travelling with a velocity of 8 and has a displacement of -4.8 from O.

Find an expression for the velocity of P at time t.

Answer

To find an expression for velocity, v, integrate the acceleration function

We can use $\int (ax+b)^n \, dx = \frac{1}{a} \cdot \frac{1}{n+1}(ax+b)^{n+1} + c$

$$v = \int (3t+2)^{-\frac{1}{3}} dt = \frac{1}{3} \times \frac{3}{2}(3t+2)^{\frac{2}{3}} + c$$

[1]

$$= \frac{1}{2}(3t+2)^{\frac{2}{3}} + c$$

To find the constant of integration, substitute $t=2$ and $v=8$the limits of integration

$$8 = \frac{1}{2}(3 \times 2 + 2)^{\frac{2}{3}} + c$$

[1]

Solve

$$8 = \frac{1}{2}(8)^{\frac{2}{3}} + c$$
$$8 = \frac{1}{2} \times 4 + c$$
$$8 = 2 + c$$
$$c = 6$$

Don't forget to write out the full expression at the end for the final mark

$$v = \frac{1}{2}(3t+2)^{\frac{2}{3}} + 6$$ [1]

(3 marks)

(b) Explain why P is never at rest.

Answer

Because the formula for the velocity is $v = \frac{1}{2}(3t+2)^{\frac{2}{3}} + 6$ and we are told that $t \geq 0$, so the velocity will always be positive [1]

In actual fact the velocity will always be greater than 6

(1 mark)

(c) Find the displacement of P from O when $t = \frac{25}{3}$.

Answer

To find an expression for displacement, s, integrate the velocity function Again, we can use $\int (ax+b)^n \, dx = \frac{1}{a} \cdot \frac{1}{n+1}(ax+b)^{n+1} + c$

$$s = \int \left(\frac{1}{2}(3t+2)^{\frac{1}{3}} + 6 \right) dt = \frac{1}{2}\left(\frac{1}{3} \times \frac{3}{5}(3t+2)^{\frac{5}{3}} \right) + 6t + c$$

[1]

$$= \frac{1}{10}(3t+2)^{\frac{5}{3}} + 6t + c$$

To find the constant of integration, substitute $t = 2$ and $s = -4.8$

$$-4.8 = \frac{1}{10}(3 \times 2 + 2)^{\frac{5}{3}} + 6 \times 2 + c$$

[1]

Solve

$$-4.8 = \frac{1}{10}(8)^{\frac{5}{3}} + 12 + c$$

$$-4.8 = \frac{1}{10} \times 32 + 12 + c$$

$$-4.8 = 3.2 + 12 + c$$

$$-4.8 = 15.2 + c$$

$$-20 = c$$

Therefore

$$s = \frac{1}{10}(3t+2)^{\frac{5}{3}} + 6t - 20$$

[1]

Substitute $t = \frac{25}{3}$ into s

$$t = \frac{25}{3},\ s = \frac{1}{10}\left(3\left(\frac{25}{3}\right)+2\right)^{\frac{5}{3}} + 6\left(\frac{25}{3}\right) - 20$$

$$= \frac{1}{10}(27)^{\frac{5}{3}} + \frac{150}{3} + 20$$

$$= \frac{1}{10} \times 243 + 50 + 20$$

$$= 24.3 + 50 - 20$$

54.3 [1]
(4 marks)

6 A particle, P, moves in a straight line such that its displacement from a fixed point at time t is s.

The acceleration of P is given by $(2t + 4)^{-\frac{1}{2}}$, for $t > 0$

(i) Given that P has a velocity of 9 when $t = 6$, find the velocity of P at time t.

(ii) Given that s = $\frac{1}{3}$ when $t = 6$, find the displacement of P at time t.

Answer

i) To find the expression for velocity, integrate the expression for acceleration. Integrate using reverse chain rule.

$$\int (2t+4)^{-\frac{1}{2}}\,dt = \frac{(2t+4)^{\frac{1}{2}}}{\frac{1}{2}} \times \frac{1}{2} + C$$

$$V = (2t+4)^{\frac{1}{2}} + C$$

[1]

To find C, substitute $t = 6$ and $V = 9$ into the equation.

$$9 = (2(6)+4)^{\frac{1}{2}} + C$$

Simplify, then rearrange, to find C.

$$9 = \sqrt{16} + C$$

$$C = 5$$

[1]

Substitute $C = 5$into the equation for velocity.

$$V = (2t+4)^{\frac{1}{2}} + 5$$ [1]

ii) To find the expression for displacement, integrate the expression for velocity.

$$\int \left((2t+4)^{\frac{1}{2}} + 5\right) dt$$

Break up the integral.

$$\int (2t+4)^{\frac{1}{2}}\,\mathrm{d}t + \int 5\,\mathrm{d}t$$

Integrate 5 with respect to t.

$$\int 5\,\mathrm{d}t = 5t + d$$

Integrate $(2t+4)^{\frac{1}{2}}$ with respect to t using reverse chain rule.

$$\int (2t+4)^{\frac{1}{2}}\,\mathrm{d}t = \frac{(2t+4)^{\frac{3}{2}}}{\frac{3}{2}} \times \frac{1}{2} + d$$

$$= \frac{1}{3}(2t+4)^{\frac{3}{2}} + d$$

Combine both results.

$$s = \frac{1}{3}(2t+4)^{\frac{3}{2}} + 5t + d$$

[1]

To find d, substitute $s = \frac{1}{3}$ and $t = 6$ into the equation.

$$\frac{1}{3} = \frac{1}{3}(2(6)+4)^{\frac{3}{2}} + 5(6) + d$$

Simplify, then rearrange, to find d.

$$\frac{1}{3} = \frac{64}{3} + 30 + d$$

$$d = -51$$

[1]

Substitute $d = -51$ into the equation for displacement.

$$\frac{1}{3}(2t+4)^{\frac{3}{2}} + 5t - 51 \text{ [1]}$$

(6 marks)

7 (a) At time ts, a particle travelling in a straight line has acceleration $(2t+1)^{-\frac{1}{2}}$ ms^{-2}. When $t = 0$, the particle is 4 m from a fixed point O and is travelling with velocity 8 ms^{-1} away from O.

Find the velocity of the particle at time t s.

Answer

To find the expression for velocity, v, integrate the expression for acceleration.

Integrate using reverse chain rule.

$$\int (2t+1)^{-\frac{1}{2}}\,\mathrm{d}t = \frac{(2t+1)^{\frac{1}{2}}}{\frac{1}{2}} \times \frac{1}{2} + C$$

$$v = (2t+1)^{\frac{1}{2}} + C$$

[1]

To find C, substitute $t = 0$ and $v = 8$ into the equation.

$$8 = (2(0)+1)^{\frac{1}{2}} + C$$

Simplify, then rearrange, to find C.

$$8 = \sqrt{1} + C$$

$$C = 7$$

[1]

Substitute $C = 7$ into the equation for velocity.

$$v = (2t+1)^{\frac{1}{2}} + 7 \text{ [1]}$$

(3 marks)

(b) Find the displacement of the particle from O at time t s.

Answer

To find the expression for displacement, s, integrate the expression for velocity.

$$\int \left((2t+1)^{\frac{1}{2}} + 7\right) \mathrm{d}t$$

Break up the integral.

$$\int (2t+1)^{\frac{1}{2}} \,\mathrm{d}t + \int 7 \,\mathrm{d}t$$

Integrate $(2t+1)^{\frac{1}{2}}$ with respect to t using reverse chain rule.

$$\int (2t+1)^{\frac{1}{2}} \,\mathrm{d}t = \frac{(2t+1)^{\frac{3}{2}}}{\frac{3}{2}} \times \frac{1}{2} + d$$

$$= \frac{1}{3}(2t+1)^{\frac{3}{2}} + d$$

Hence,

$$s = \frac{1}{3}(2t+1)^{\frac{3}{2}} + 7t + d$$

for an attempt to integrate answer to part a[1] *for correct result* [1]

To find d, substitute $s = 4$ and $t = 0$ into the equation.

$$4 = \frac{(2(0)+1)^{\frac{3}{2}}}{3} + 7(0) + d$$

Simplify, then rearrange, to find d.

$$4 = \frac{1}{3} + d$$

$$d = \frac{11}{3}$$

[1]

Substitute $d = \frac{11}{3}$ into the equation for displacement.

$$s = \frac{1}{3}(2t+1)^{\frac{3}{2}} + 7t + \frac{11}{3}$$ [1]

(4 marks)

8 (a) A particle travels along a straight line with a velocity, v ms^{-1}, given by

$$v = 6\left(t - \frac{1}{2}\right)^2 + \frac{5}{2}$$

for $t \geq 0$ where t is time in seconds. The particle has an initial displacement of -1 metres.

Find the acceleration after 2 seconds.

Answer

Differentiate velocity to find acceleration

Method 1

Expand and simplify the velocity first

$$v = 6\left(t^2 - \frac{1}{2}t - \frac{1}{2}t + \frac{1}{4}\right) + \frac{5}{2}$$
$$v = 6\left(t^2 - t + \frac{1}{4}\right) + \frac{5}{2}$$
$$v = 6t^2 - 6t + \frac{3}{2} + \frac{5}{2}$$
$$v = 6t^2 - 6t + 4$$

[M1]

Then differentiate v to find a

$$a = 12t - 6$$

[M1 A1]

Substitute in $t = 2$

$$a = 12 \times 2 - 6$$

18 ms^{-2}

[A1]

Method 2

Use the chain rule, $a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}t}$ where $u = t - \frac{1}{2}$

$$v = 6u^2 + \frac{5}{2} \qquad u = t - \frac{1}{2}$$

$$\frac{\mathrm{d}v}{\mathrm{d}u} = 12u \qquad \frac{\mathrm{d}u}{\mathrm{d}t} = 1$$

[M1]

This gives

$$a = 12u \times 1$$

$$a = 12\left(t - \frac{1}{2}\right)$$

[A1 A1]

Mark Scheme and Guidance

A1: For an answer at least in the form $k\left(t - \frac{1}{2}\right)$.

A1: For $12\left(t - \frac{1}{2}\right)$ or $12t - 6$.

Substitute in $t = 2$

$$a = 12\left(2 - \frac{1}{2}\right)$$

18 ms^{-2}

[A1]

(4 marks)

(b) Find the displacement after 2 seconds.

Answer

Method 1

Integrate $v = 6t^2 - 6t + 4$ to get displacement s and add a constant of integration

$$s = \frac{6t^3}{3} - \frac{6t^2}{2} + 4t + c$$

[M1]

Mark Scheme and Guidance

This mark is for at least two correct terms in t (can be unsimplified).

Simplify

$$s = 2t^3 - 3t^2 + 4t + c$$

[A1]

The initial displacement is -1 so substitute in $s = -1$ when $t = 0$ to find c

$$-1 = 0 - 0 + 0 + c$$
$$c = -1$$

Substitute this value of c back into the expression for s

$$s = 2t^3 - 3t^2 + 4t - 1$$

[A1]

Substitute in $t = 2$

$$s = 2(2)^3 - 3(2)^2 + 4(2) - 1$$

11 m

[A1]

Method 2

Integrate $v = 6\left(t - \frac{1}{2}\right)^2 + \frac{5}{2}$ using $\int (ax+b)^n \, dx = \frac{1}{a} \times \frac{1}{n+1}(ax+b)^{n+1} + c$ to get displacement s

Add a constant of integration

$$s = 6 \times \frac{1}{1} \times \frac{1}{3}\left(t - \frac{1}{2}\right)^3 + \frac{5}{2}t + c$$

[M1 A1]

Simplify

$$s = 2\left(t - \frac{1}{2}\right)^3 + \frac{5}{2}t + c$$

The initial displacement is -1 so substitute in $s = -1$ when $t = 0$ to find c

$$-1 = 2\left(0 - \frac{1}{2}\right)^3 + 0 + c$$

$$-1 = -\frac{1}{4} + c$$

$$c = -\frac{3}{4}$$

Substitute this value of c back into the expression for s

$$s = 2\left(t - \frac{1}{2}\right)^3 + \frac{5}{2}t - \frac{3}{4}$$

[A1]

Substitute in $t = 2$

$$s = 2\left(2 - \frac{1}{2}\right)^3 + \frac{5}{2}(2) - \frac{3}{4}$$

11 m

[A1]

(4 marks)

(c) Explain why a velocity-time graph and a speed-time graph will always be the same for this particle.

Answer

Speed is the positive part of the velocity, $|v|$

Method 1

Use algebra to show that $v = 6\left(t - \frac{1}{2}\right)^2 + \frac{5}{2}$ is always positive

Start by noting that $\left(t - \frac{1}{2}\right)^2 \geq 0$ as it is a squared expression

$$\left(t - \frac{1}{2}\right)^2 \geq 0$$
$$6\left(t - \frac{1}{2}\right)^2 \geq 0$$
$$6\left(t - \frac{1}{2}\right)^2 + \frac{5}{2} > 0$$

This expression is actually positive for all t values, but you need just $t \geq 0$

$v = 6\left(t - \frac{1}{2}\right)^2 + \frac{5}{2}$ **is always positive for** $t \geq 0$ **so the velocity-time graph and speed-time graph are the same**

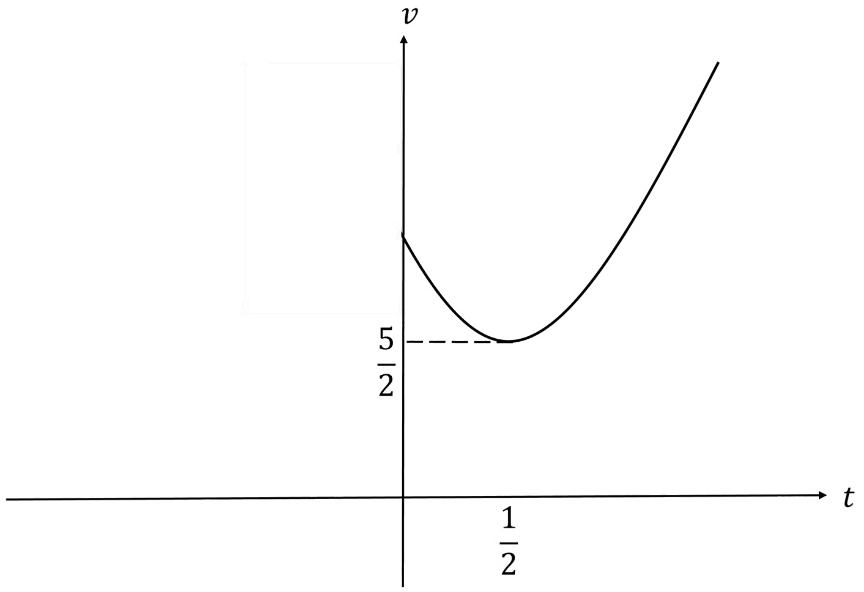
[B1]

Mark Scheme and Guidance

Your answer must refer to the fact that $v = 6\left(t - \frac{1}{2}\right)^2 + \frac{5}{2}$ is always positive for $t \geq 0$ (or all t) to score the mark.

Method 2

The velocity-time graph of $v = 6\left(t - \frac{1}{2}\right)^2 + \frac{5}{2}$ is a U-shaped quadratic with a minimum point of $\left(\frac{1}{2}, \frac{5}{2}\right)$ above the t-axis



$v = 6\left(t - \frac{1}{2}\right)^2 + \frac{5}{2}$ is always positive for $t \geq 0$ so the velocity-time graph and speed-time graph are the same

[B1]

Mark Scheme and Guidance

Your answer must refer to $v = 6\left(t - \frac{1}{2}\right)^2 + \frac{5}{2}$ always being positive for $t \geq 0$ (or all t) to score the mark.

The graph sketch is not necessary for the mark.

Examiner Tips and Tricks

Even though the graph sketch is not awarded any marks here, it is still a good idea to include it.

(1 mark)

Very Hard Questions

1 (a) A particle moves in a straight line such that, t seconds after passing a fixed point O, its displacement from O is s m, where $s = e^{2t} - 10e^{t} - 12t + 9$.

Find expressions for the velocity and acceleration at time t.

Answer

Velocity is the change in speed with respect to time, therefore differentiate to get

$$v = \frac{ds}{dt} = 2e^{2t} - 10e^{t} - 12$$

$2e^{2t}$ [1]

fully correct expression [1]

Acceleration is the change in velocity with respect to time, therefore we differentiate to get

$$a = \frac{dv}{dt} = 4e^{2t} - 10e^{t}$$ [1]

(3 marks)

(b) Find the time when the particle is instantaneously at rest.

Answer

If the particle is at rest, this means that the velocity is 0.

$$0 = 2e^{2t} - 10e^{t} - 12$$

This is a hidden quadratic which is easier to solve if we let $e^{t} = x$

$$0 = 2x^2 - 10x - 12$$

Dividing through by 2 gives

$$0 = x^2 - 5x - 6$$

We can factorise this by looking for a factor pair of −6 that has a sum of −5. This would be −6 and +1.

$$0 = (x - 6)(x + 1)$$

[1]

This means that the solutions are

$$x = 6 \Rightarrow e^t = 6$$

[1]

$x = -1 \Rightarrow e^t = -1$ which has no solutions

Taking natural log of both sides of the first solution gives

$t = \ln 6$ (seconds) [1]

(3 marks)

(c) Find the acceleration at this time.

Answer

Substituting $t = \ln 6$ into the equation for a gives

$$a = 4e^{2\ln 6} - 10e^{\ln 6}$$

correct substitution [1]

$a = 84$ (ms^{-2}) [1]

(2 marks)