IGCSE · Cambridge (CIE) · Further Maths

4 hours · 32 questions

Exam Questions

Differentiation

Introduction to Differentiation / Differentiating Special Functions / Chain Rule / Product Rule / Quotient Rule / Applications of Differentiation / Second Order Derivatives / Modelling with Differentiation / Connected Rates of Change

Medium (12 questions)	/77
Hard (14 questions)	/102
Very Hard (6 questions)	/45
Total Marks	**/224**

Medium Questions

1 Given that $y = \tan x$, use calculus to find the approximate change in y as x increases from $-\frac{\pi}{4}$ to $h - \frac{\pi}{4}$, where h is small.

(3 marks)

2 Find the x-coordinate of the stationary point on the curve $y = \left(2 - \sqrt{3}\right)x^2 + x - 1$, giving your answer in the form $a + b\sqrt{3}$,where a and b are rational numbers.

(3 marks)

3 The radius, r cm, of a circle is increasing at the rate of 5 cms^{-1}. Find, in terms of π, the rate at which the area of the circle is increasing when $r = 3$.

(4 marks)

4 The volume, V, of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$. The radius, r cm, of a sphere is increasing at the rate of 0.5 cms^{-1}. Find, in terms of π, the rate of change of the volume of the sphere when $r = 0.25$.

(4 marks)

5 (a) Given that $y=(x^2-1)\sqrt{5x+2}$, show that $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{Ax^2+Bx+C}{2\sqrt{5x+2}}$, where A, B and C are integers.

(5 marks)

(b) Find the coordinates of the stationary point of the curve $y=(x^2-1)\sqrt{5x+2}$ for $x>0$. Give each coordinate correct to 2 significant figures.

(3 marks)

(c) Determine the nature of this stationary point.

(2 marks)

6 Variables x and y are such that $y = \sin x + \mathrm{e}^{-x}$. Use differentiation to find the approximate change in y as x increases from $\frac{\pi}{4}$ to $\frac{\pi}{4}+h$, where h is small.

(4 marks)

7 (a) The equation of a curve is $y = x\sqrt{16-x^2}$ for $0 \leq x \leq 4$.

Find the exact coordinates of the stationary point of the curve.

(6 marks)

(b) Find $\frac{\mathrm{d}}{\mathrm{d}x}(16-x^2)^{\frac{3}{2}}$ and hence evaluate the area enclosed by the curve $y = x\sqrt{16-x^2}$ and the lines $y = 0$, $x = 1$ and $x = 3$.

(5 marks)

8 (a) A curve has equation $y = (2x - 1)\sqrt{4x + 3}$.

Show that $\dfrac{dy}{dx} = \dfrac{4(Ax + B)}{\sqrt{4x + 3}}$, where A and B are constants.

(5 marks)

(b) Hence write down the x-coordinate of the stationary point of the curve.

(1 mark)

(c) Determine the nature of this stationary point.

(2 marks)

9 (a) Find the equation of the tangent to the curve $y = x^3 - 6x^2 + 3x + 10$ at the point where $x = 1$.

(4 marks)

(b) Find the coordinates of the point where this tangent meets the curve again.

(5 marks)

10 (a) It is given that $y = \ln(1 + \sin x)$ for $0 < x < \pi$.

Find $\frac{dy}{dx}$

(2 marks)

(b) Find the value of $\frac{dy}{dx}$ when $x = \frac{\pi}{6}$, giving your answer in the form $\frac{1}{\sqrt{a}}$, where a is an integer.

(2 marks)

(c) Find the values of x for which $\frac{dy}{dx} = \tan x$.

(5 marks)

11 (a) A curve has equation $y = x \cos x$.

Find $\frac{dy}{dx}$.

(2 marks)

(b) Find the equation of the normal to the curve at the point where $x = \pi$, giving your answer in the form $y = mx + c$.

(4 marks)

12 Find the equation of the tangent to the curve $y = \frac{\ln(3x^2 - 1)}{x + 2}$ at the point where $x = 1$.

Give your answer in the form $y = mx + c$, where m and c are constants correct to 3 decimal places.

(6 marks)

Hard Questions

1 A curve has equation $y = \ln(5 - 3x)$ where $x < \frac{5}{3}$. The normal to the curve at the point where $x = -5$, cuts the x-axis, at the point P.
Find the equation of the normal and the x-coordinate of P.

(7 marks)

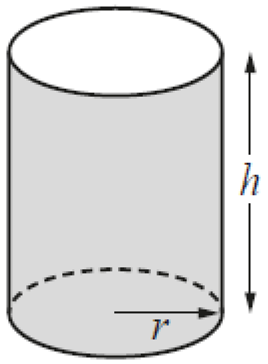
2 Variables x and y are such that $y = e^{\frac{x}{2}} + x\cos 2x$, where x is in radians. Use differentiation to find the approximate change in y as x increases from 1 to $1 + h$, where h is small.

(6 marks)

3 The tangent to the curve $y = \ln(3x^2 - 4) - \frac{x^3}{6}$, at the point where $x = 2$, meets the y-axis at the point P. Find the exact coordinates of P.

(6 marks)

4



A container is a circular cylinder, open at one end, with a base radius of r cm and a height of h cm. The volume of the container is 1000 cm^3. Given that r and h can vary and that the total outer surface area of the container has a minimum value, find this value.

(8 marks)

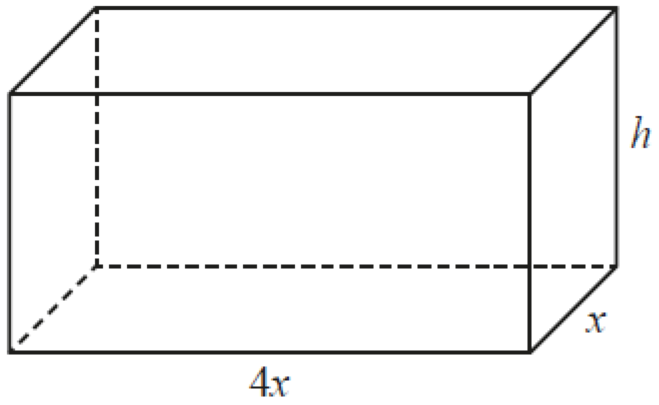
5 (a) Find the x-coordinates of the stationary points of the curve $y = e^{3x}(2x+3)^6$.

(6 marks)

(b) A curve has equation $y = \mathrm{f}(x)$ and has exactly two stationary points. Given that $\mathrm{f}''(x) = 4x - 7$, $\mathrm{f}'(0.5) = 0$ and $\mathrm{f}'(3) = 0$, use the second derivative test to determine the nature of each of the stationary points of this curve.

(2 marks)

(c) In this question all lengths are in centimetres.



The diagram shows a solid cuboid with height h and a rectangular base measuring $4x$ by x. The volume of the cuboid is 40 cm^3. Given that x and h can vary and that the surface area of the cuboid has a minimum value, find this value.

(5 marks)

6 (a) Find the equation of the tangent to the curve $2y = \tan 2x + 7$ at the point where $x = \frac{\pi}{8}$.

Give your answer in the form $ax - y = \frac{\pi}{b} + c$, where a, b and c are integers.

(5 marks)

(b) This tangent intersects the x-axis at P and the y-axis at Q. Find the length of PQ.

(2 marks)

7 (a) $y = x\sqrt{x+2}$ Given that , show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{Ax+B}{2\sqrt{x+2}}$, where A and B are constants.

(5 marks)

(b) Find the exact coordinates of the stationary point of the curve $y = x\sqrt{x+2}$.

(3 marks)

(c) Determine the nature of this stationary point.

(2 marks)

8 (a) Differentiate $y = \tan(x+4) - 3\sin x$ with respect to x.

(2 marks)

(b) Variables x and y are such that $y = \dfrac{\ln(2x+5)}{2e^{3x}}$. Use differentiation to find the approximate change in y as x increases from 1 to $1 + h$, where h is small.

(6 marks)

9 (a) It is given that $y = \dfrac{\tan 3x}{\sin x}$.

Find the exact value of $\dfrac{dy}{dx}$ when $x = \dfrac{\pi}{3}$.

(4 marks)

(b) Hence find the approximate change in y as x increases from $\dfrac{\pi}{3}$ to $\dfrac{\pi}{3} + h$, where h is small.

(1 mark)

(c) Given that x is increasing at the rate of 3 units per second, find the corresponding rate of change in y when $x = \dfrac{\pi}{3}$, giving your answer in its simplest surd form.

(2 marks)

10 (a) It is given that $y = \ln(\sin x + 3\cos x)$ for $0 < x < \frac{\pi}{2}$.

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(3 marks)

(b) Find the value of x for which $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2}$.

(3 marks)

11 (a) Given that $y = \dfrac{e^{2x-3}}{x^2+1}$, find $\dfrac{dy}{dx}$.

(3 marks)

(b) Hence, given that y is increasing at the rate of 2 units per second, find the exact rate of change of x when $x = 2$.

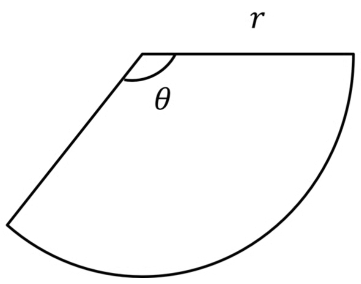
(3 marks)

12 A sphere of radius r cm and volume V cm^3 is increasing in size with time t seconds. The volume increases at a constant rate of 24 cm^3 s^{-1}.

Find the exact rate at which the radius is increasing when the sphere reaches a volume of $\dfrac{32\pi}{3}$ cm^3.

(4 marks)

13 (a) A sector from a circle of radius r has an internal angle of θ radians, as shown below.



The perimeter of the sector is 4 units and the area of the sector is A square units.

Show that $A = \dfrac{8\theta}{(2+\theta)^2}$.

(3 marks)

(b) Show that $\dfrac{\mathrm{d}A}{\mathrm{d}\theta} = \dfrac{p(q-\theta)}{(2+\theta)^3}$ where p and q are constants to be found.

(3 marks)

(c) Find the maximum area of the sector. You must show that this area is a maximum.

(4 marks)

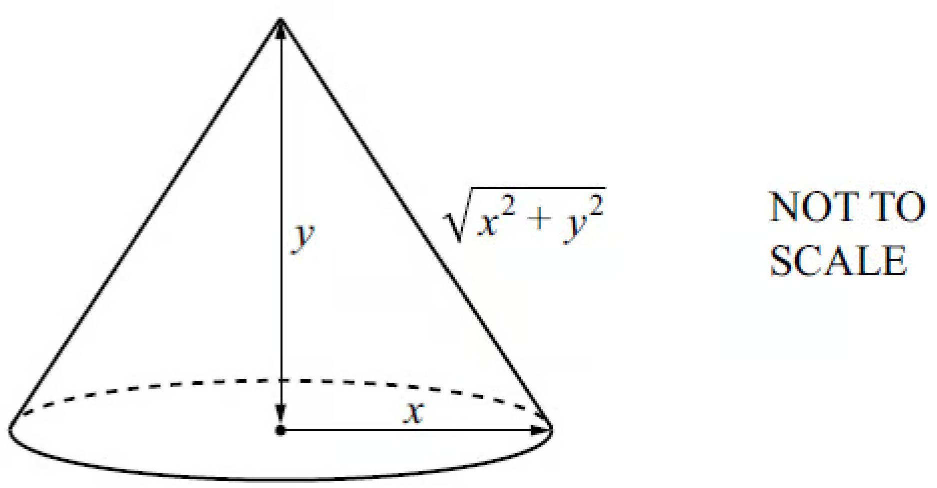
14 A curve is given by $y = \ln(1 + x^4)$.

Use calculus to find the approximate change in y as x increases from -1 to $-1 + k$ where k is small.

(4 marks)

Very Hard Questions

1 (a) In this question, all lengths are in centimetres.



The diagram shows a cone of base radius x, height y and sloping edge $\sqrt{x^2 + y^2}$. The volume of the cone is $10\pi\ \text{cm}^3$.

Show that the curved surface area, S, of the cone is given by $S = \dfrac{\pi\sqrt{x^6 + 900}}{x}$.

(3 marks)

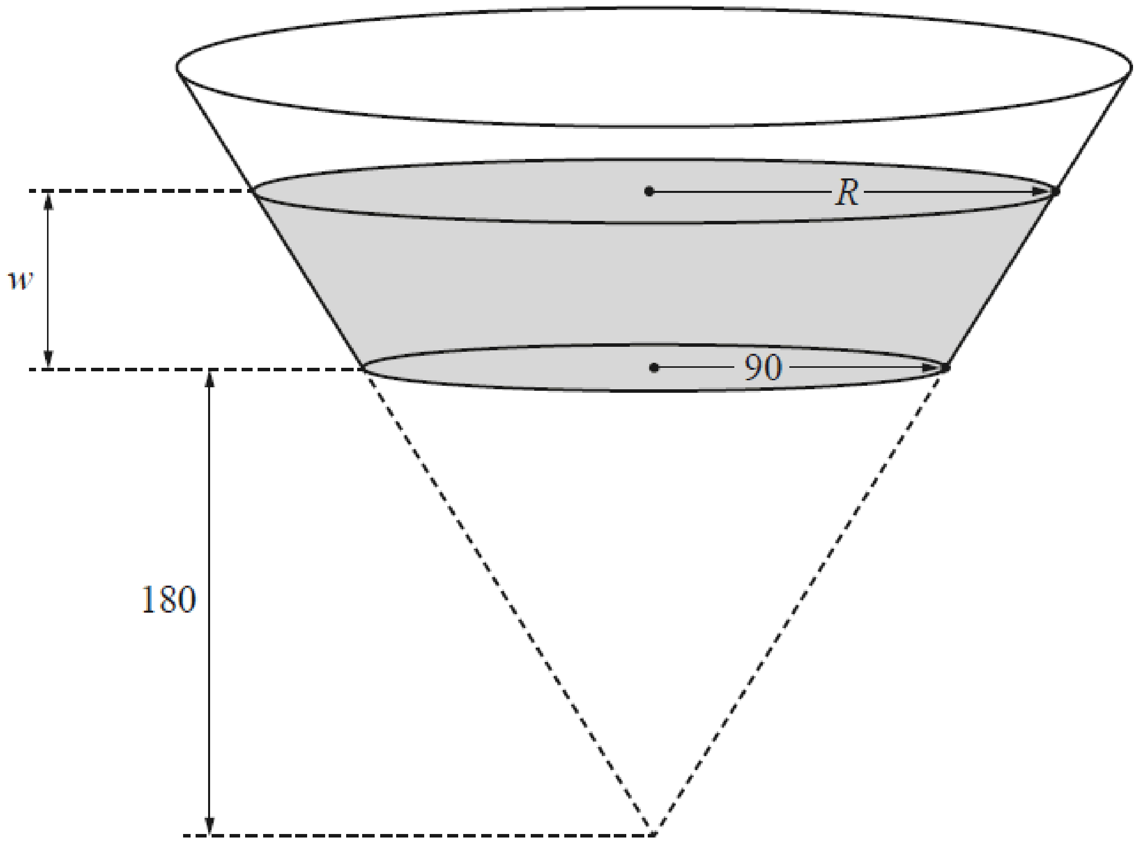
(b) Given that x can vary and that S has a minimum value, find the value of x for which S is a minimum.

(5 marks)

2 Variables x and y are such that $y=\dfrac{e^{3x}\sin x}{x^2}$ .Use differentiation to find the approximate change in y as x increases from 0.5 to $0.5+h$, where h is small.

(6 marks)

3 (a) In this question all lengths are in centimetres. The volume, V, of a cone of height h and base radius r is given by $V = \frac{1}{3}\pi r^2 h$



The diagram shows a large hollow cone from which a smaller cone of height 180 and base radius 90 has been removed. The remainder has been fitted with a circular base of radius 90 to form a container for water. The depth of water in the container is w and the surface of the water is a circle of radius R.

Find an expression for R in terms of w and show that the volume V of the water in the container is given by $V = \frac{\pi}{12}(w + 180)^3 - 486000\pi$.

(3 marks)

(b) Water is poured into the container at a rate of $10\,000\ \text{cm}^3\text{s}^{-1}$. Find the rate at which the depth of the water is increasing when $w = 10$.

(4 marks)

4 (a) A curve has equation $y = \dfrac{\ln(3x^2 - 5)}{2x + 1}$ for $3x^2 > 5$

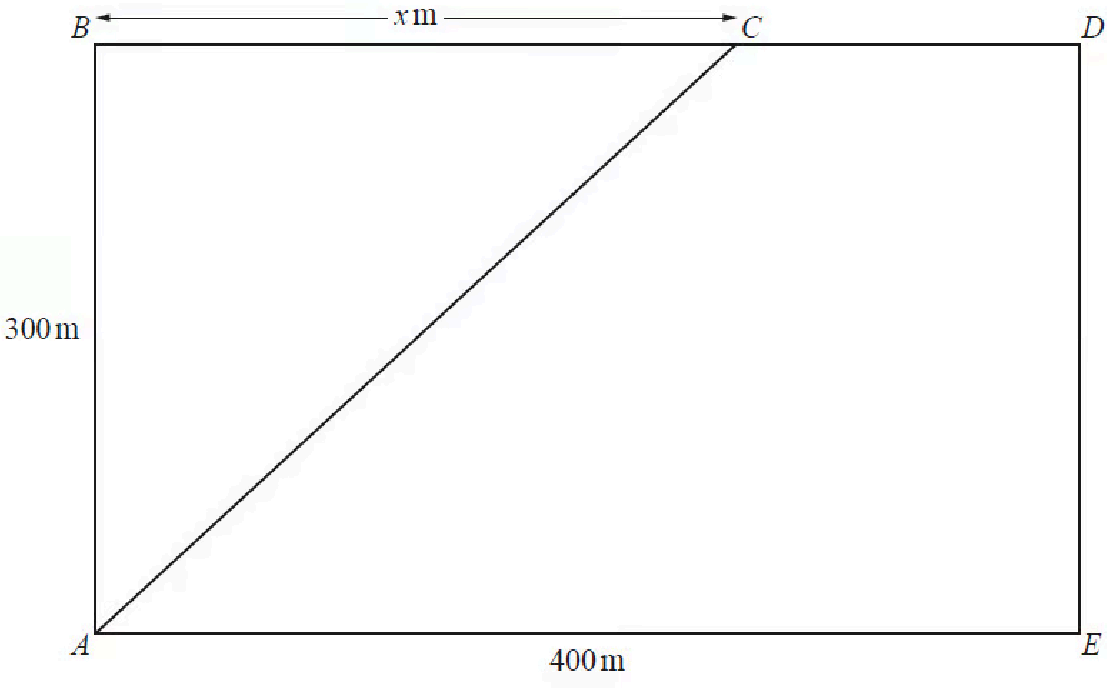
Find the equation of the normal to the curve at the point where $x = \sqrt{2}$.

(6 marks)

(b) Find the approximate change in y as x increases from $\sqrt{2}$ to $\sqrt{2} + h$, where h is small.

(1 mark)

5 (a)



The rectangle $ABCDE$ represents a ploughed field where $AB = 300\text{ m}$ and $AE = 400\text{ m}$. Joseph needs to walk from A to D in the least possible time. He can walk at 0.9 ms^{-1} on the ploughed field and at 1.5 ms^{-1} on any part of the path BCD along the edge of the field. He walks from A to C and then from C to D. The distance $BC = x\text{ m}$.

Find, in terms of x, the total time, $T\text{ s}$, Joseph takes for the journey.

(3 marks)

(b) Given that x can vary, find the value of x for which T is a minimum and hence find the minimum value of T.

(6 marks)

6 (a) A curve has the equation $y = e^{(4x - x^2)}$.

Find and factorise an expression for $\frac{dy}{dx}$.

(2 marks)

(b) Hence find $\int \frac{(2 - x)e^{4x}}{e^{x^2}} dx$.

(2 marks)

(c) Show that the second derivative of the curve $y = e^{(4x - x^2)}$ satisfies the relationship

$$\frac{d^2y}{dx^2} = (p + qx + rx^2)y$$

where p, q and r are constants to be found.

(4 marks)