IGCSE · Cambridge (CIE) · Further Maths   2 hours   19 questions

Exam Questions

Integration

Introduction to Integration / Integrating Powers of x / Definite Integrals / Finding Areas with Integration / Finding Areas Between Lines & Curves / Integrating Trig Functions / Integrating e^x & 1/x / Reverse Chain Rule

Medium (3 questions)	/18
Hard (11 questions)	/86
Very Hard (5 questions)	/44
Total Marks	**/148**

Medium Questions

1 (a) Giving your answer in its simplest form, find the exact value of

$$\int_0^4 \frac{10}{5x+2}\,dx$$

(4 marks)

(b) $\int_0^{\ln 2} (e^{4x} + 2)^2 \, dx$

(5 marks)

2 Find $\frac{d}{dx}(16 - x^2)^{\frac{3}{2}}$ and hence evaluate the area enclosed by the curve $y = x\sqrt{16 - x^2}$ and the lines $y = 0$, $x = 1$ and $x = 3$.

(5 marks)

3 Find $\int \frac{1}{(7x+4)^m} \mathrm{d}x$ in the following cases.

(a) $m = 2$

(b) $m = 1$

(4 marks)

Hard Questions

1 Find the exact value of $\int_2^4 \frac{(x+1)^2}{x^2}\,\mathrm{d}x$.

(6 marks)

2 Given

$$\frac{\mathrm{d}}{\mathrm{d}x}(x\cos x) = -x\sin x + \cos x$$

find the exact value of $\int_0^{\frac{\pi}{6}} x\sin x\,\mathrm{d}x$.

(5 marks)

3 (a) Given that $\int_1^a \left(\frac{2}{2x+3} + \frac{3}{3x-1} - \frac{1}{x}\right)\mathrm{d}x = \ln 2.4$ where $a > 1$, find the value of a.

(7 marks)

(b) (i) Find $\frac{\mathrm{d}}{\mathrm{d}x}(6 \sin^3 kx)$, where k is a constant.

(ii) Hence find $\int(\sin^2 2x \cos 2x)dx.$

(4 marks)

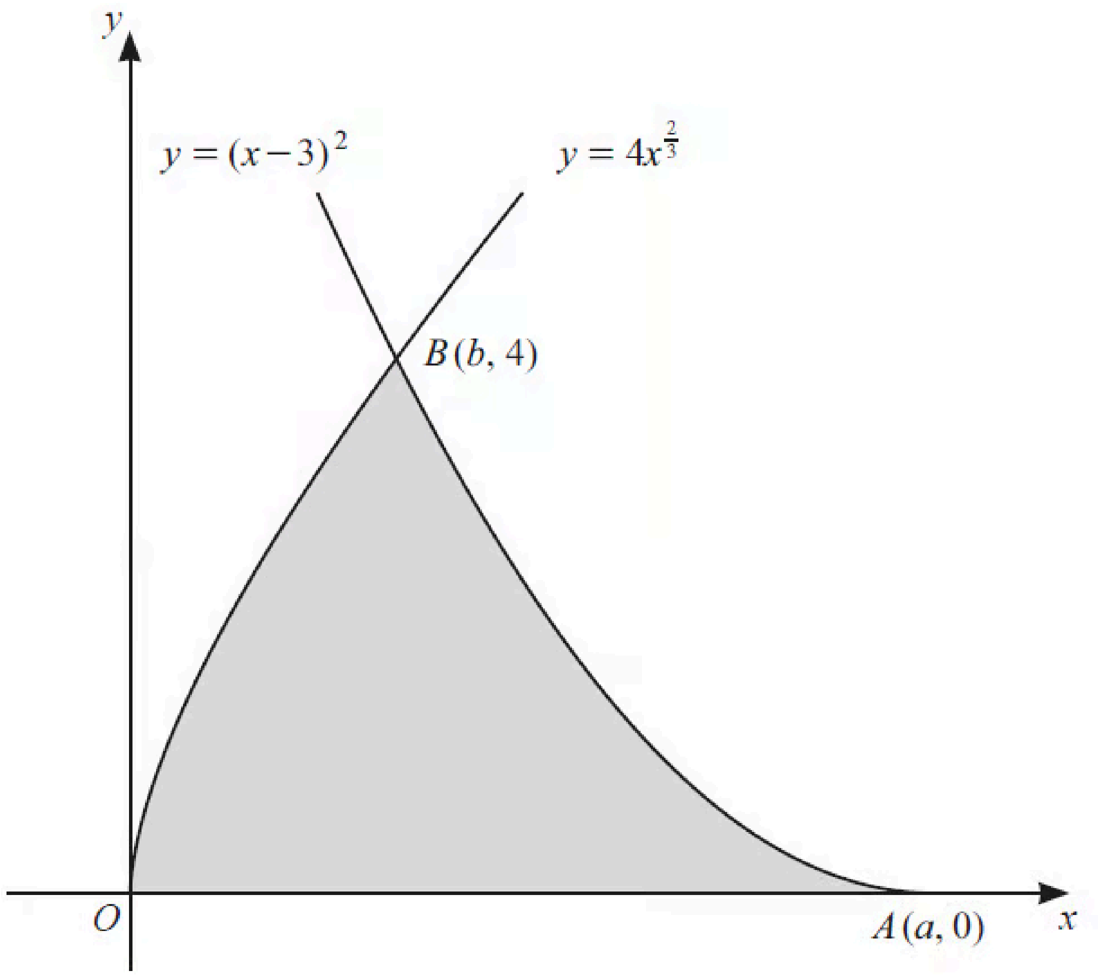
4 Find $\int_3^5 \left(\frac{1}{x-1} - \frac{1}{(x-1)^2}\right)\mathrm{d}x$, giving your answer in the form $a + \ln b$, where a and b are rational numbers.

(5 marks)

5 A curve is such that $\frac{d^2y}{dx^2} = \sin\left(6x - \frac{\pi}{2}\right)$. Given that $\frac{dy}{dx} = \frac{1}{2}$ at the point $\left(\frac{\pi}{4}, \frac{13\pi}{12}\right)$ on the curve, find the equation of the curve.

(7 marks)

6 (a)



The diagram shows part of the graphs of $y=4x^{\frac{2}{3}}$ and $y = (x-3)^2$ .The graph of $y = (x-3)^2$ meets the x-axis at the point $A(a, 0)$ and the two graphs intersect at the point $B(b, 4)$.

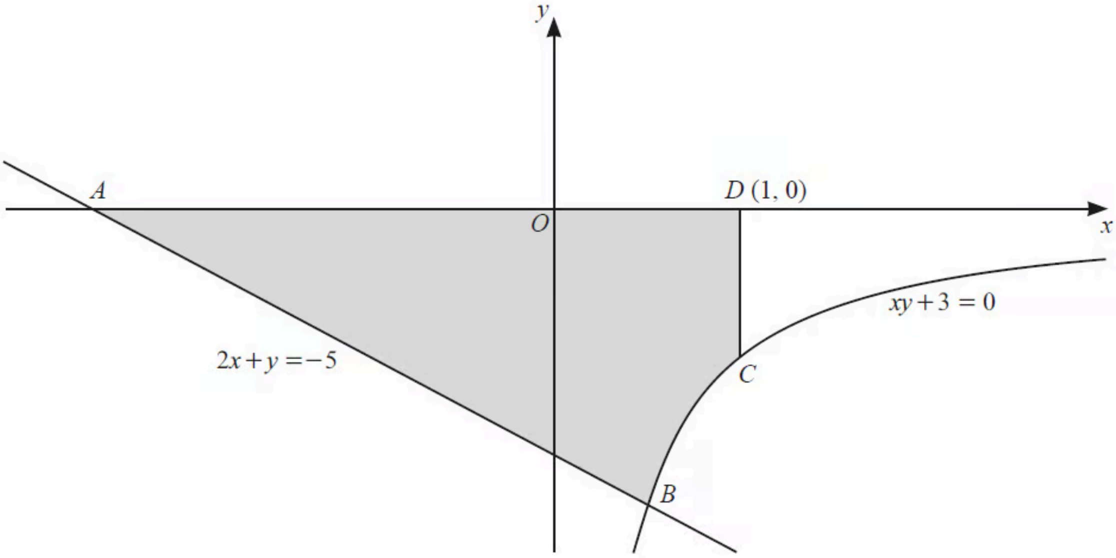
Find the value of a and of b.

(2 marks)

(b) Find the area of the shaded region.

(5 marks)

7 (a)



The diagram shows the straight line $2x + y = -5$ and part of the curve $xy + 3 = 0$. The straight line intersects the x-axis at the point A and intersects the curve at the point B. The point C lies on the curve. The point D has coordinates (1, 0). The line CD is parallel to the y-axis.

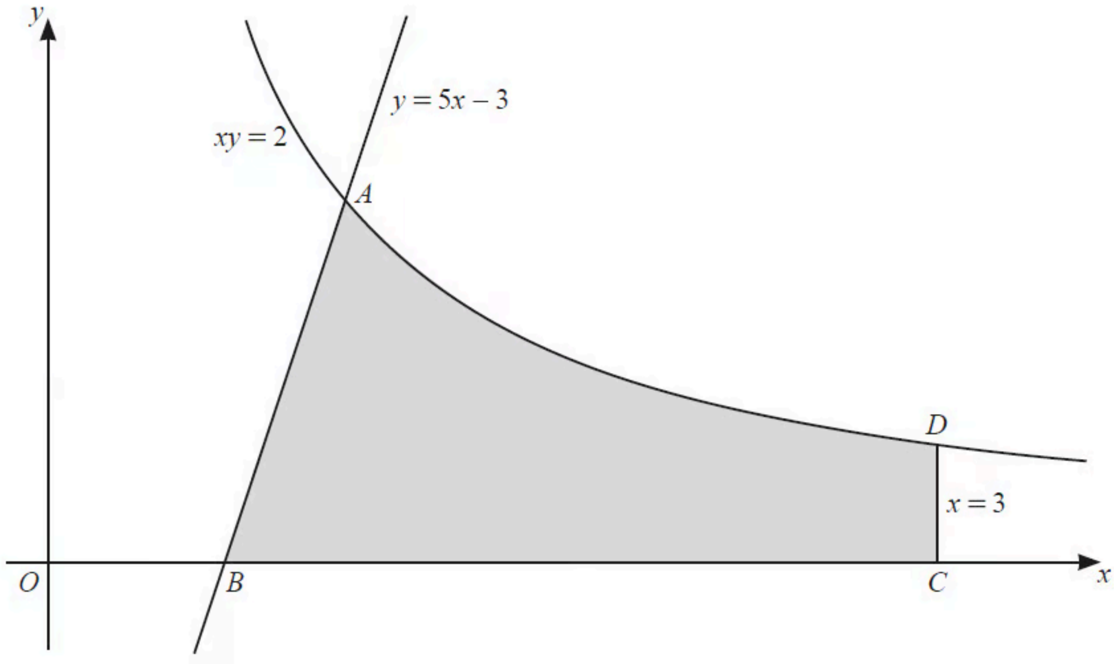
Find the coordinates of each of the points A and B.

(3 marks)

(b) Find the area of the shaded region, giving your answer in the form $p + \ln q$, where p and q are positive integers.

(6 marks)

8



The diagram shows part of the curve $xy = 2$ intersecting the straight line $y = 5x - 3$ at the point A.
The straight line meets the x-axis at the point B. The point C lies on the x-axis and the point D lies on the curve such that the line CD has equation $x = 3$. Find the exact area of the shaded region, giving your answer in the form $p + \ln q$, where p and q are constants.

(8 marks)

9 (a) (i) Given that $\mathrm{f}(x) = \dfrac{1}{\cos x}$, show that $\mathrm{f}'(x) = \tan x \sec x$.

[3]

(ii) Hence find $\int\left(3 \tan x \sec x - \sqrt[4]{e^{3x}}\right) \mathrm{d}x$.

[3]

(6 marks)

(b) Given that $\int_2^5 \frac{p}{px + 10} \mathrm{d}x = \ln 2$, find the value of the positive constant p.

(5 marks)

10 (a) Given that $\int_1^a \left(\frac{1}{x} - \frac{1}{2x+3}\right) dx = \ln 3$, where $a > 0$, find the exact value of a, giving your answer in simplest surd form.

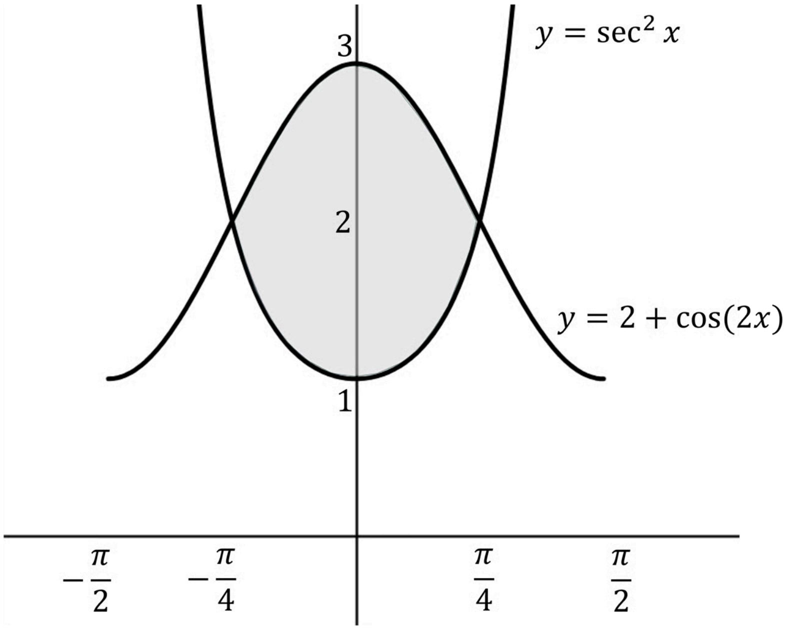
(6 marks)

(b) Find the exact value of $\int_0^{\frac{\pi}{3}} \left(\sin\left(2x + \frac{\pi}{3}\right) - 1 + \cos 2x\right) dx$.

(5 marks)

11 Throughout this question, x is measured in radians.

The curves $y = \sec^2 x$ and $y = 2 + \cos(2x)$ intersect at the points $\left(-\frac{\pi}{4}, 2\right)$ and $\left(\frac{\pi}{4}, 2\right)$, as shown.



Find the exact area of the shaded region enclosed.

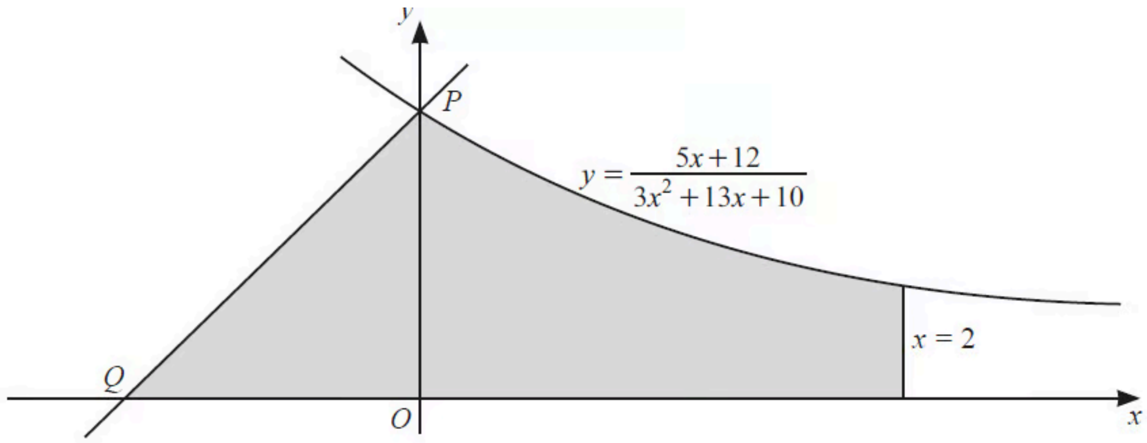
(6 marks)

Very Hard Questions

1 (a) Show that $\frac{1}{x+1}+\frac{2}{3x+10}$ can be written as $\frac{5x+12}{3x^2+13x+10}$

(1 mark)

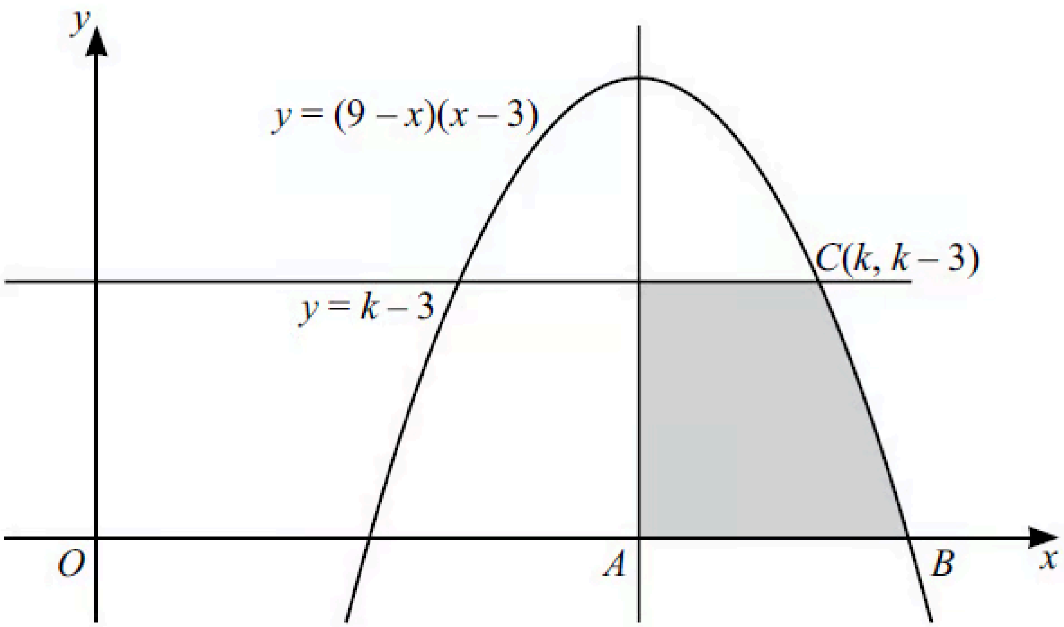
(b)



The diagram shows part of the curve $y=\frac{5x+12}{3x^2+13x+10}$, the line $x=2$ and a straight line of gradient 1. The curve intersects the y-axis at the point P. The line of gradient 1 passes through P and intersects the x-axis at the point Q. Find the area of the shaded region, giving your answer in the form $a+\frac{2}{3}\ln\left(b\sqrt{3}\right)$, where a and b are constants.

(9 marks)

2



The diagram shows part of the curve $y = (9 - x)(x - 3)$ and the line $y = k - 3$, where $k > 3$.
The line through the maximum point of the curve, parallel to the y-axis, meets the x-axis at A.
The curve meets the x-axis at B, and the line $y = k - 3$ meets the curve at the point $C(k, k - 3)$.

Find the area of the shaded region.

(9 marks)

3 (a) Show that $\frac{3}{2x-3}+\frac{3}{2x+3}$ can be written as $\frac{12x}{4x^2-9}$.

(2 marks)

(b) Hence find $\int \frac{12x}{4x^2-9}\mathrm{d}x$, giving your answer as a single logarithm and an arbitrary constant.

(3 marks)

(c) Given that $\int_2^a \frac{12x}{4x^2-9}\mathrm{d}x = \ln 5\sqrt{5}$, where $a > 2$, find the exact value of a.

(4 marks)

4 A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=5\cos 2x$. This curve has a gradient of $\frac{3}{4}$ at the point $\left(-\frac{\pi}{12},\frac{5\pi}{4}\right)$. Find the equation of this curve.

(8 marks)

5 (a) The gradient of the normal to a curve at the point (x, y) is given by $\frac{x}{x+1}$

Given that the curve passes through the point $(1, 4)$, show that its equation is $y = 5 - \ln x - x$.

(5 marks)

(b) Find, in the form $y = mx + c$, the equation of the tangent to the curve at the point where $x = 3$.

(3 marks)