IGCSE · Cambridge (CIE) · Further Maths

47 mins   7 questions

Exam Questions

Straight-Line Graphs

Linear Graphs / Parallel & Perpendicular Lines

Medium (2 questions)	/8
Hard (2 questions)	/15
Very Hard (3 questions)	/24
Total Marks	**/47**

Medium Questions

1 Variables x and y are such that, when $\sqrt[4]{y}$ is plotted against $\frac{1}{x}$, a straight line graph passing through the points (0.5, 9) and (3, 34) is obtained. Find y as a function of x.

Answer

Using the gradient formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, we have

$$m = \frac{34 - 9}{3 - 0.5} = 10$$

[1]

Using the formula for the equation of a line and substituting one of the co-ordinates

$$Y - 9 = 10(X - 0.5)$$

[1]

Expanding the brackets

$$Y - 9 = 10X - 5$$

$$Y = 10X + 4$$

We are looking for when $\sqrt[4]{y}$ is plotted against $\frac{1}{x}$ so we substitute these into the equation

$$\sqrt[4]{y} = 10\left(\frac{1}{x}\right) + 4$$

$$\sqrt[4]{y} = \frac{10}{x} + 4$$

[1]

Raising each side to the power of 4

$$y = \left(\frac{10}{x} + 4\right)^4 \text{ [1]}$$

(4 marks)

2 Solutions to this question by accurate drawing will not be accepted.

Find the equation of the perpendicular bisector of the line joining the points $(4, -7)$ and $(-8, 9)$.

Answer

Find the midpoint of the line segment.

$$\left(\frac{4-8}{2}, \frac{9-7}{2}\right)$$

$$\text{Midpoint} = (-2, 1)$$

[1]

Find the gradient of the line.

$$\text{gradient} = \frac{9 - -7}{-8-4}$$

$$\text{gradient} = -\frac{4}{3}$$

[1]

Find the gradient of the perpendicular to the line by taking the negative reciprocal.

$$\text{gradient of perpendicular} = \frac{3}{4}$$

[1]

Substitute this gradient and the coordinates of the midpoint into the equation $y - y_1 = m(x - x_1)$

$$y - 1 = \frac{3}{4}(x - -2)$$

Expand and simplify.

$y = \frac{3}{4}x + \frac{5}{2}$ [1]

(4 marks)

Hard Questions

1 (a) **Solutions by accurate drawing will not be accepted.** The points A and B have coordinates (-2, 4) and (6,10) respectively.

Find the equation of the perpendicular bisector of the line AB, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Answer

Find the midpoint of AB

$$\text{Midpoint} = \left(\frac{-2+6}{2}, \frac{4+10}{2}\right)$$

$$\text{Midpoint} = (2, 7)$$

[1]

To find the equation of the perpendicular line, first find the gradient of AB

$$\text{Gradient}_{AB} = \frac{10-4}{6--2}$$

$$\text{Gradient}_{AB} = \frac{6}{8} = \frac{3}{4}$$

[1]

Now use the negative reciprocal to find the gradient of the perpendicular bisector

$$\text{Gradient}_{perp} = -\frac{4}{3}$$

Now we know a point on the line and the gradient, so find the equation of the perpendicular bisector

$$y - 7 = -\frac{4}{3}(x-2)$$

[1]

Multiply the equation by 3

$$3y - 21 = -4(x - 2)$$

Expand the bracket

$$3y - 21 = -4x + 8$$

Make the equation equal to 0

$4x + 3y - 29 = 0$ [1]

(4 marks)

(b) The point C has coordinates $(5, p)$ and lies on the perpendicular bisector of AB. Find the value of p.

Answer

From part (a), the equation of the perpendicular bisector is $4x + 3y - 29 = 0$

Find y when $x = 5$ by substituting into the equation

$$(4 \times 5) + 3y - 29 = 0$$

$$20 + 3y - 29 = 0$$

$$3y = 9$$

$$y = 3$$

3 [1]

(1 mark)

(c) It is given that the line AB bisects the line CD.
Find the coordinates of D.

Answer

Let M = midpoint of *CD*

If *AB* bisects *CD*, then $\overrightarrow{CM} = \overrightarrow{MD}$

Find $\overrightarrow{CM}$

$$C = (5, 3) \text{ and } M = (2, 7)$$

$$\overrightarrow{CM} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

[1]

$$\text{So } \overrightarrow{MD} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

Apply this vector from point M to get D

$$D = (-1, 11)$$

$(-1, 11)$ [1]

(2 marks)

2 (a) Find the equation of the perpendicular bisector of the line joining the points (12, 1) and (4, 3), giving your answer in the form $y = mx + c$.

Answer

Find the gradient, m, of the line joining the points $(12, 1)$ and $(4, 3)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{3 - 1}{4 - 12}$$

$$m = -\frac{1}{4}$$

[1]

Find the gradient of the perpendicular line, m_p, by using the negative reciprocal of m

$$m_p = 4$$

[1]

Find the midpoint, M, of the 2 points

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M = \left(\frac{12 + 4}{2}, \frac{1 + 3}{2}\right)$$

$$M = (8, 2)$$

[1]

Find the equation of the perpendicular line using its gradient and midpoint

$$y - 2 = 4(x - 8)$$

[1]

Rearrange to give the form $y = mx + c$ as required

$$y - 2 = 4x - 32$$

$$y = 4x - 30$$

$y = 4x - 30$ [1]

(5 marks)

(b) The perpendicular bisector cuts the axes at points A and B. Find the length of AB.

Answer

From part (a) the equation of the perpendicular bisector is $y = 4x - 30$

The line will cross the x-axis when $y = 0$

Substitute $y = 0$ into the equation and solve for x

$$0 = 4x - 30$$

$$4x = 30$$

$$x = 7.5$$

[1]

The line will cross the y-axis when $x = 0$

Substitute $x = 0$ into the equation and solve for y

$$y = 4(0) - 30$$

$$y = -30$$

[1]

The 2 points we have are $(7.5, 0)$ and $(0, -30)$

Find the distance between the points using the distance formula

$$d = \sqrt{(7.5^2 + 0^2) + \sqrt{0^2 + (-30)^2}}$$

30.9 [1]

(3 marks)

Very Hard Questions

1 (a) **Solutions to this question by accurate drawing will not be accepted.** The points A and B are (4, 3) and (12, −7) respectively.

Find the equation of the line L, the perpendicular bisector of the line AB.

Answer

Find the midpoint, M, of AB

$$M=\left(\frac{4+12}{2},\ \frac{3+(-7)}{2}\right)$$

$$M=(8,\ -2)$$

[1]

Calculate the gradient between points A and B and simplify

$$\text{gradient}_{AB}=\frac{-7-3}{12-4}$$

$$\text{gradient}_{AB}=-\frac{5}{4}$$

[1]

Find the gradient of the perpendicular line, L, by finding the negative reciprocal of $-\frac{5}{4}$

$$\text{gradient}_{L}=\frac{4}{5}$$

[1]

Find the equation of the line through the midpoint, $M=(8,\ -2)$ with gradient $\frac{4}{5}$

$$y-(-2)=\frac{4}{5}(x-8)$$

$$y+2=\frac{4}{5}(x-8)$$ [1]

Equivalent forms are accepted such as $4x-5y=42$

(4 marks)

(b) The line parallel to AB which passes through the point (5, 12) intersects L at the point C. Find the coordinates of C.

Answer

The line parallel to *AB* has the same gradient as the line *AB*

From part (a), $\text{gradient}_{AB} = -\frac{5}{4}$

$$\text{gradient}_{parallel\ line} = -\frac{5}{4}$$

The parallel line passes through point $(5, 12)$

Find the equation of the line

$$y-12=-\frac{5}{4}(x-5)$$

[1]

Rearrange to make y the subject

(3) $$y=-\frac{5}{4}(x-5)+12$$

From part (a), the equation of line L is

$$y+2=\frac{4}{5}(x-8)$$

Rearrange to make y the subject

$$(4) \qquad y = \frac{4}{5}(x-8) - 2$$

Set the equations (3) and (4) equal and solve them

$$-\frac{5}{4}(x-5) + 12 = \frac{4}{5}(x-8) - 2$$

[1]

Solve for x

Multiply through by 20 (multiply by 4 and then by 5)

$$-25(x-5) + 240 = 16(x-8) - 40$$

Expand the brackets and solve

$$-25x + 125 + 240 = 16x - 128 - 40$$

$$-25x + 365 = 16x - 168$$

$$365 = 41x - 168$$

$$x = 13$$

[1]

Substitute $x = 13$ into equation (4) to find y

$$y = \frac{4}{5}(13-8) - 2$$

$$y = 2$$

Write the answer as coordinates

$(13, 2)$ [1]

(4 marks)

2 The curves $y = x^2$ and $y^2 = 27x$ intersect at O(0, 0) and at the point A. Find the equation of the perpendicular bisector of the line OA.

Answer

To find where the curves intersect at point A, substitute $y = x^2$ into $y^2 = 27x$

$$(x^2)^2 = 27x$$

[1]

$$x^4 - 27x = 0$$

[1]

Factorise and solve.

$$x(x^3 - 27) = 0$$

[1]

$$x = 0$$

$$x = 3$$

We already know there is an intersection at $(0, 0)$. Substitute $x = 3$ into one of the given equations to find the corresponding y coordinate.

$$y = (3)^2$$

$$y = 9$$

Therefore, point A is

$$(3, 9)$$

[1]

We need to find a perpendicular bisector, therefore we need to find the midpoint of line OA.

$$\left(\frac{0+3}{2}, \frac{0+9}{2}\right)$$

$$\text{Midpoint} = \left(\frac{3}{2}, \frac{9}{2}\right)$$

[1]

The gradient of OA is

$$\frac{9-0}{3-0}$$

$$= 3$$

[1]

Therefore, the gradient of the perpendicular to OA is

$$-\frac{1}{3}$$

[1]

Substitute into $y - y_1 = m(x - x_1)$ and rearrange to find the equation of the perpendicular bisector.

$$y - \frac{9}{2} = -\frac{1}{3}\left(x - \frac{3}{2}\right)$$

$y = -\frac{1}{3}x + 5$ [1]

(8 marks)

3 (a) The line $y = 5x + 6$ meets the curve $xy = 8$ at the points A and B.

Find the coordinates of A and of B.

Answer

Substituting the equation of the line into the equation of the curve and solving these equations simultaneously by substitution

$$x(5x + 6) = 8$$

Expanding the brackets and setting the equation equal to 0

$$5x^2 + 6x - 8 = 0$$

[1]

Factorising to find the solutions

$$(5x - 4)(x + 2) = 0$$

$$x = \frac{4}{5},\ x = -2$$

Substituting to find the *y*- coordinates

$$y = 5\left(\frac{4}{5}\right) + 6 = 10$$

$$y = 5(-2) + 6 = -4$$

$\left(\frac{4}{5}, 10\right)$ [1]

$(-2, -4)$ [1]

(3 marks)

(b) Find the coordinates of the point where the perpendicular bisector of the line AB meets the line $y = x$.

Answer

The perpendicular bisector meets the line AB at the midpoint of AB

$$\left(\frac{\frac{4}{5} - 2}{2}, \frac{10 - 4}{2}\right)$$

$$\left(-\frac{3}{5}, 3\right)$$

[1]

The equation of the line is in the form $y = mx + c$ so we know

$$m = 5$$

[1]

We will use the negative reciprocal to find the perpendicular gradient and substitute into the formula $y - y_1 = m(x - x_1)$

$$y - 3 = -\frac{1}{5}\left(x + \frac{3}{5}\right)$$

[1]

Substituting $y = x$ and solving

$$x - 3 = -\frac{1}{5}\left(x + \frac{3}{5}\right)$$

[1]

Expanding the brackets and collecting like terms gives

$$\frac{6}{5}x - 3 = -\frac{3}{25}$$

$$\frac{6}{5}x = \frac{72}{25}$$

$$x = \frac{12}{5}$$

The equation of the line is $y = x$, therefore the y- coordinate will be the same

$\left(\frac{12}{5}, \frac{12}{5}\right)$ [1]

(5 marks)