IGCSE · Cambridge (CIE) · Further Maths

2 hours · 27 questions

Exam Questions

Trigonometry

Trigonometric Functions / The Unit Circle / Graphs of Trigonometric Functions / Trigonometric Identities / Solving Trigonometric Equations / Trigonometric Proof

Medium (9 questions)	/37
Hard (9 questions)	/44
Very Hard (9 questions)	/62
Total Marks	**/143**

Medium Questions

1 (a) In this question, all angles are in radians.

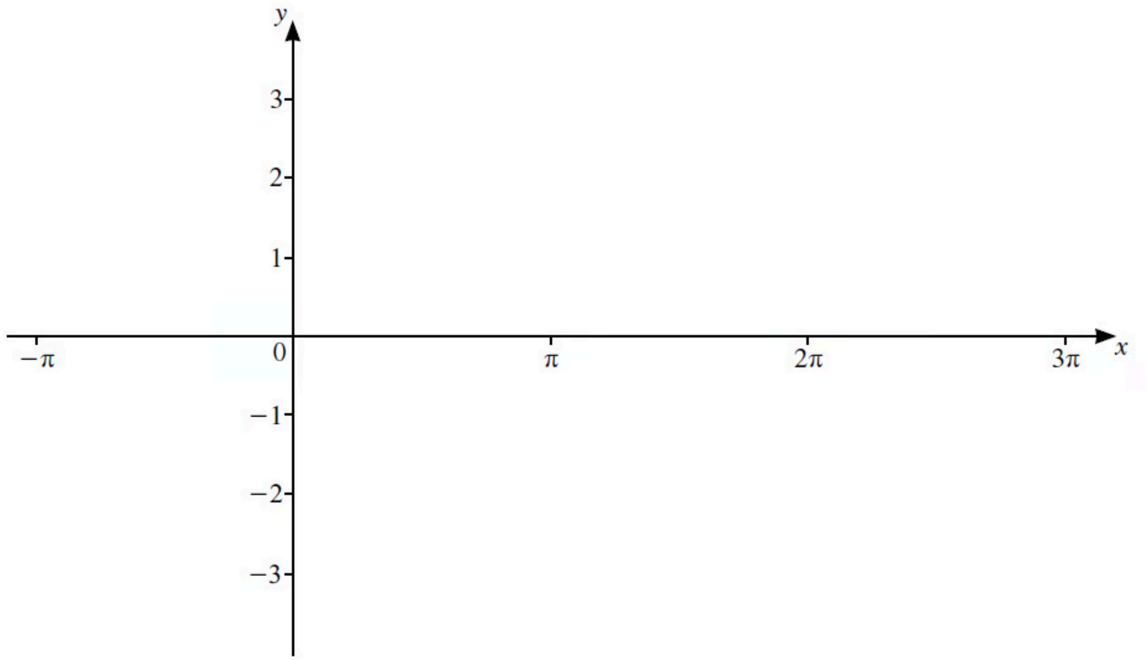
Write down the amplitude of $2\cos\frac{x}{3} - 1$.

(1 mark)

(b) Write down the period of $2\cos\frac{x}{3} - 1$.

(2 marks)

(c) On the axes below, sketch the graph of $y = 2\cos\frac{x}{3} - 1$ for $-\pi \leqslant x \leqslant 3\pi$.



(3 marks)

2 (i) Write $6xy + 3y + 4x + 2$ in the form $(ax + b)(cy + d)$, where a, b, c and d are positive integers.

(ii) Hence solve the equation $6 \sin \theta \cos \theta + 3 \cos \theta + 4 \sin \theta + 2 = 0$ for $0° < \theta < 360°$.

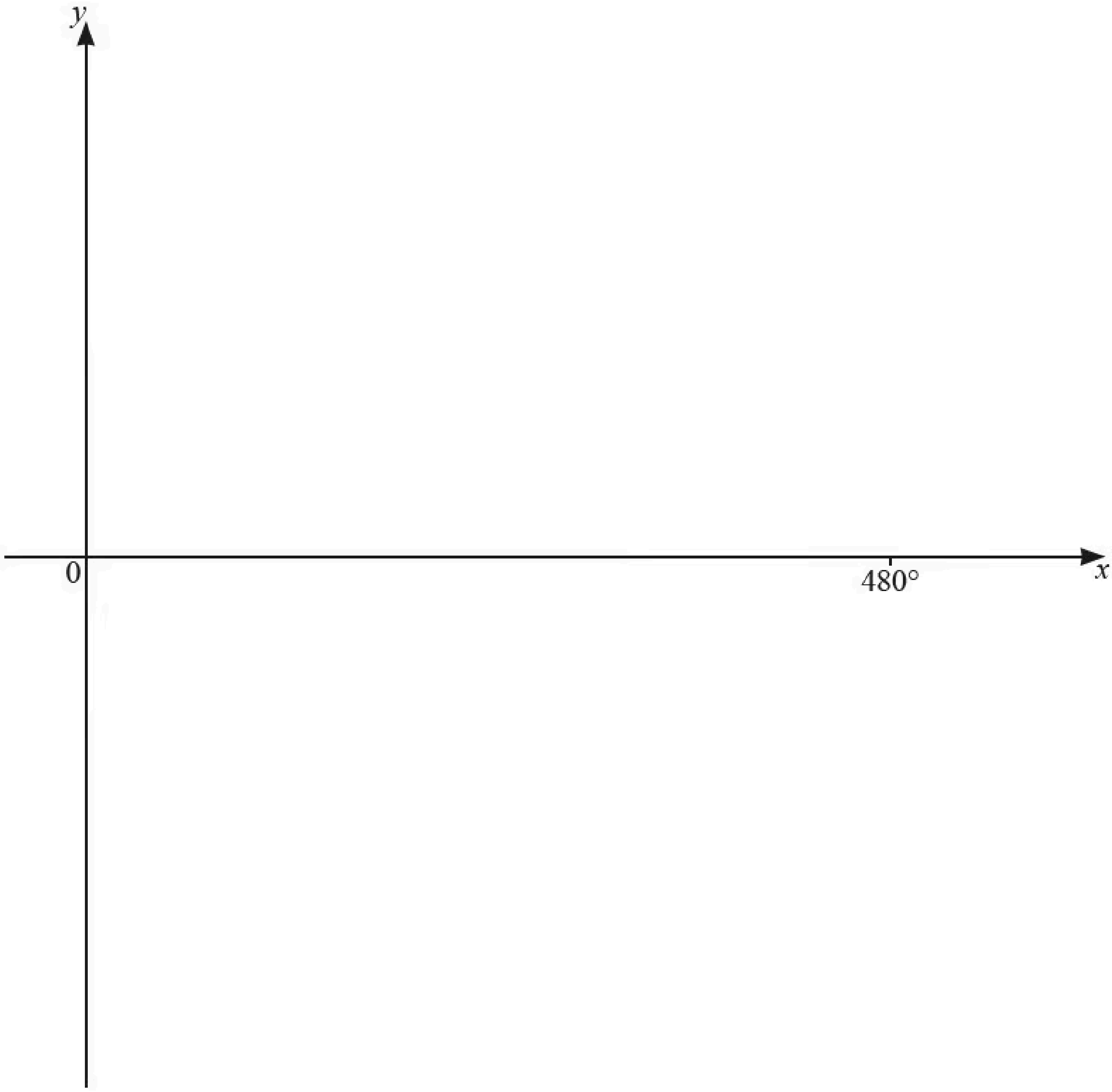
(5 marks)

3 (a) The graph of $y = a + 2 \tan bx$, where a and b are constants, passes through the point $(0, -4)$ and has period $480°$.

Find the value of a and of b.

(3 marks)

(b) On the axes, sketch the graph of y for values of x between 0° and 480°.

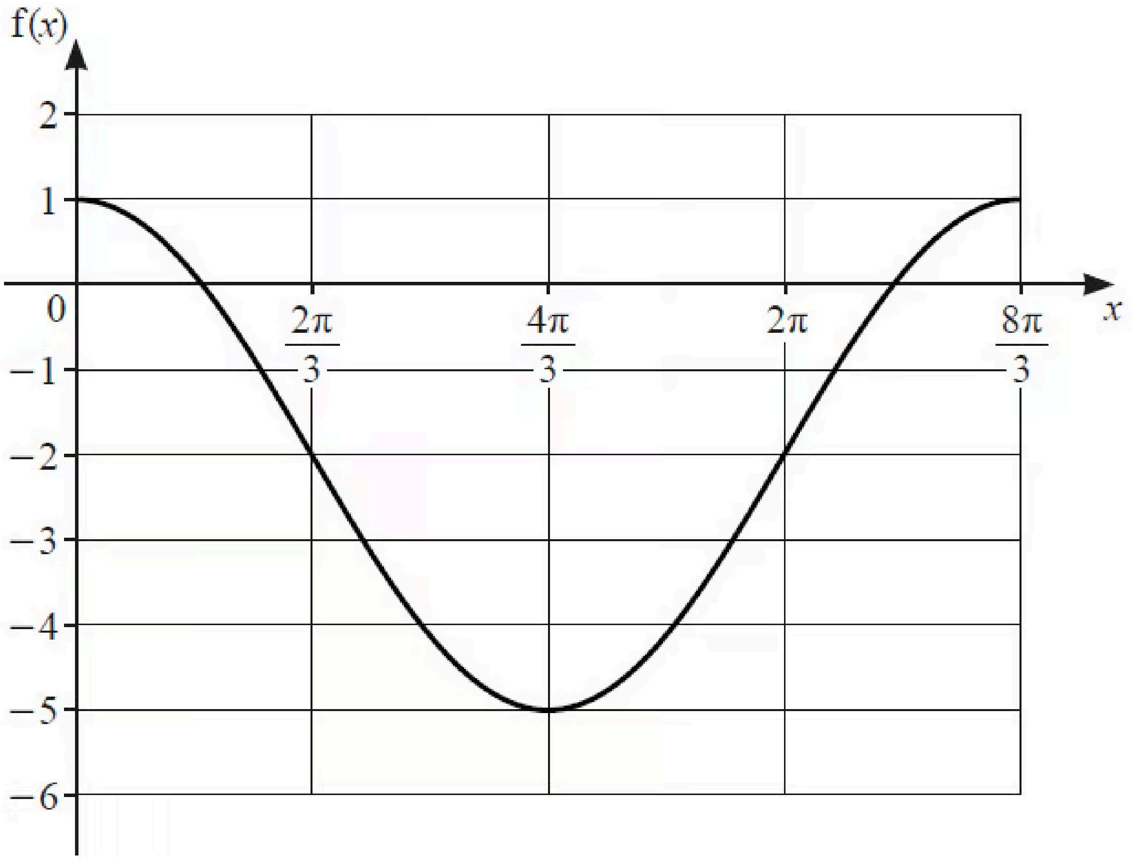


(2 marks)

4 Solve $\tan(\alpha + 45^\circ) = -\dfrac{1}{\sqrt{2}}$ for $0^\circ \le \alpha \le 360^\circ$.

(3 marks)

5



The diagram shows the graph of $f(x) = a\cos bx + c$ for $0 \le x \le \frac{8\pi}{3}$ radians.

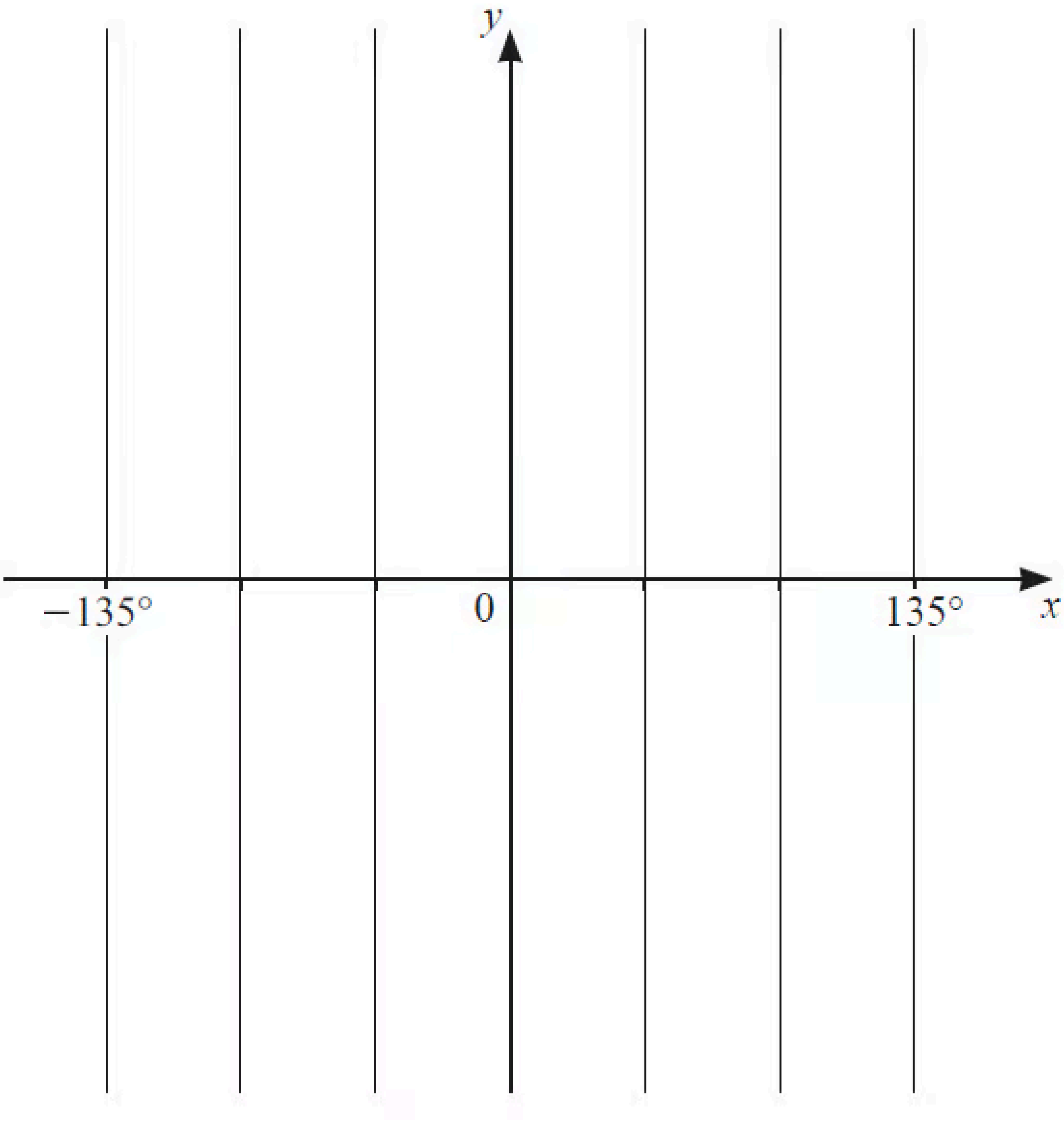
Find the value of each of the constants a, b and c.

(4 marks)

6 The curve $y = a\sin bx + c$ has a period of 180°, an amplitude of 20 and passes through the point $(90°, -3)$. Find the value of each of the constants a, b and c.

(3 marks)

7 The function g is defined, for $-135° \leq x \leq 135°$, by $g(x) = 3\tan\frac{x}{2} - 4$. Sketch the graph of $y = g(x)$ on the axes below, stating the coordinates of the point where the graph crosses the y-axis.



(2 marks)

8 Solve the equation $5\sin\left(4B-\frac{\pi}{8}\right)+2=0$ for $-\frac{\pi}{4}\leq B\leq\frac{\pi}{4}$ radians

(4 marks)

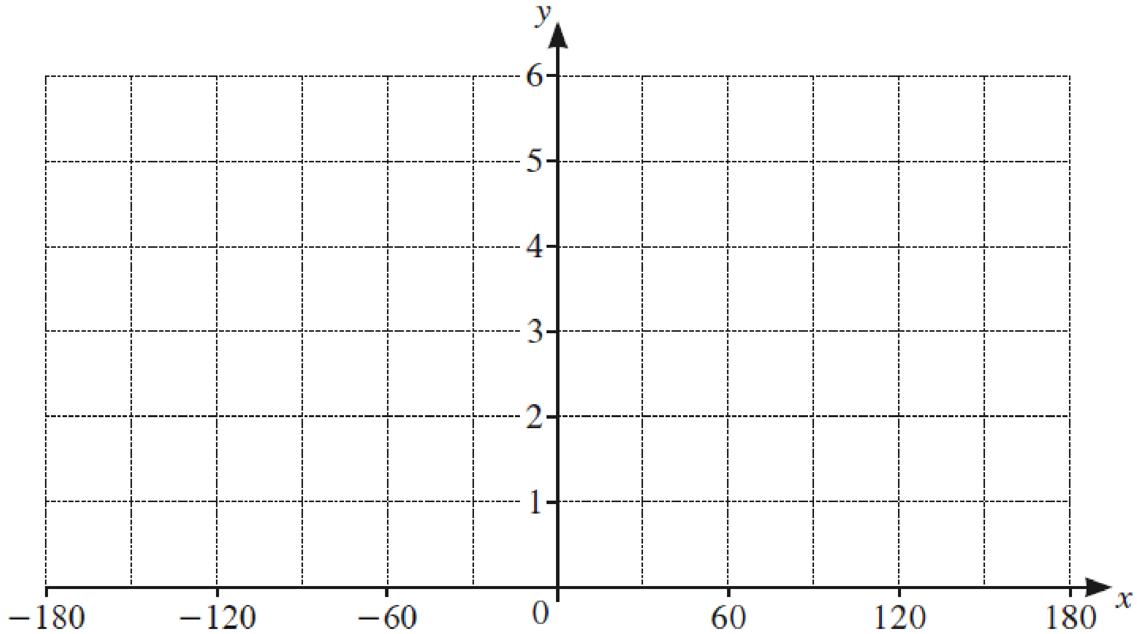
9 (a) Write down the amplitude of $1+4\cos\left(\frac{x}{3}\right)$

(1 mark)

(b) Write down the period of $1+4\cos\left(\frac{x}{3}\right)$

(1 mark)

(c) On the axes below, sketch the graph of $y=1+4\cos\left(\frac{x}{3}\right)$ for $-180°\leq x°\leq 180°$.



(3 marks)

Hard Questions

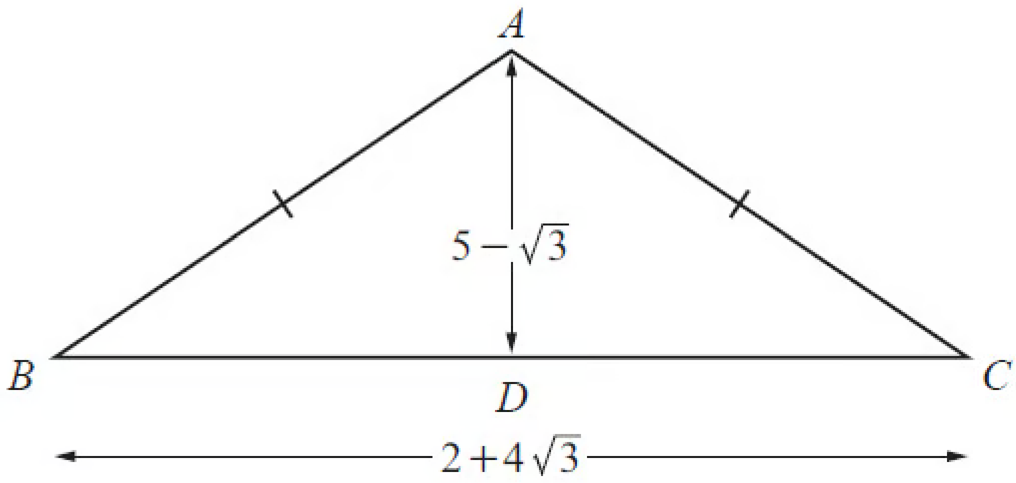
1 Solve the equation $\cot\left(y - \frac{\pi}{2}\right) = \sqrt{3}$, where y is in radians and $0 \leqslant y \leqslant \pi$.

(3 marks)

2 Solve the equation $\frac{1}{2}\sec\left(2\phi + \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}}$ for $-\pi < \phi < \pi$, where ϕ is in radians. Give your answers in terms of π.

(5 marks)

3 (a) In this question all lengths are in centimetres.



The diagram shows the isosceles triangle ABC, where $AB = AC$ and $BC = 2 + 4\sqrt{3}$.
The height, AD, of the triangle is $5 - \sqrt{3}$.

Find tan ABC, giving your answer in the form $c + d\sqrt{3}$, where c and d are integers.

(2 marks)

(b) Find $\sec^2 ABC$, giving your answer in the form $e + f\sqrt{3}$, where e and f are integers.

(2 marks)

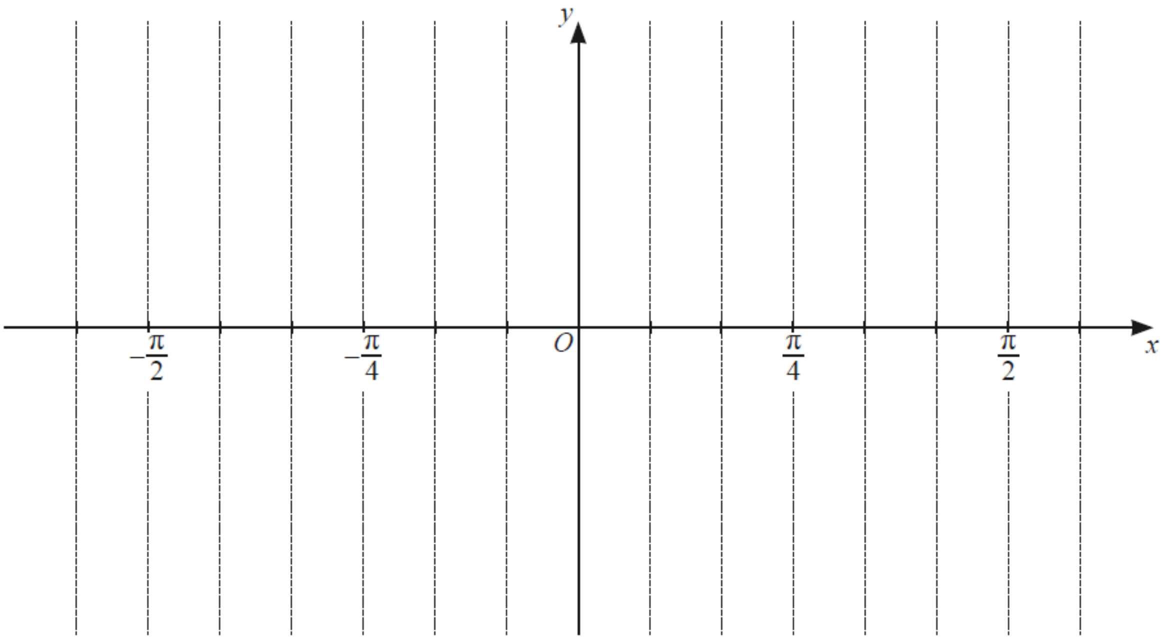
4 Solve $\operatorname{cosec}\left(y + \frac{\pi}{3}\right) = 2 \text{ for } 0 \leq y \leq 2\pi$ radians, giving your answers in terms of π.

(4 marks)

5 (a) Solve $\tan 3x = -1$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ radians, giving your answers in terms of π.

(4 marks)

(b) Use your answers to part (a) to sketch the graph of $y = 4\tan 3x + 4$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ radians on the axes below. Show the coordinates of the points where the curve meets the axes.



(3 marks)

6 Solve $3\cot^2 x - 14 \operatorname{cosec} x - 2 = 0$ for $0° < x < 360°$.

(5 marks)

7 Solve the equation

$$5\sec^2 A + 14\tan A - 8 = 0 \text{ for } 0° \le A \le 180°,$$

(4 marks)

8 Solve $\sin\left(3\phi + \frac{2\pi}{3}\right) = \cos\left(3\phi + \frac{2\pi}{3}\right)$ for $0 \le \phi \le \frac{2\pi}{3}$ radians, giving your answers in terms of π.

(4 marks)

9 (a) In this question, all angles are measured in radians.

The graph of $y = a\sin bx + c$ has an amplitude of 10, a period of 16π and passes through the point with coordinates $(12\pi, 4)$.

Find the constants a, b and c.

(4 marks)

(b) Sketch the graph of $y = \left|\tan\left(\frac{x}{2}\right) + 1\right|$ for $-\pi < x < \pi$.

Label clearly any points of intersection with the coordinate axes and any asymptotes.

(4 marks)

Very Hard Questions

1 (i) Show that $\dfrac{\cos^2 2x}{1+\sin 2x} = 1 - \sin 2x$.

(ii) Hence solve the equation for $\dfrac{3\cos^2 2x}{1+\sin 2x} = 1$ for $0° \leqslant x \leqslant 90°$.

(6 marks)

2 (i) Show that $\dfrac{1}{\sin\theta - 1} - \dfrac{1}{\sin\theta + 1} = a\sec^2\theta$, where a is a constant to be found.

[3]

(ii) Hence solve $\dfrac{1}{\sin 3\phi - 1} - \dfrac{1}{\sin 3\phi + 1} = -8$ for $-\dfrac{\pi}{3} \leq \phi \leq \dfrac{\pi}{3}$ radians.

[5]

(8 marks)

3 (i) Show that $\frac{1}{\sec\theta - 1} - \frac{1}{\sec\theta + 1} = 2\cot^2\theta$.

[3]

(ii) Hence solve $\frac{1}{\sec 2x - 1} - \frac{1}{\sec 2x + 1} = 6$ for $-90° < x < 90°$.

[5]

(8 marks)

4 Show that $\frac{\sin^4 y - \cos^4 y}{\cot y} = \tan y - 2\cos y \sin y$.

(4 marks)

5 (i) Show that $\dfrac{1}{(1+\operatorname{cosec}\theta)(\sin\theta-\sin^2\theta)} = \sec^2\theta$.

[4]

(ii) Hence solve $(1+\operatorname{cosec}\theta)(\sin\theta-\sin^2\theta) = \dfrac{3}{4}$ for $-180° \le \theta \le 180°$.

[4]

(8 marks)

6 Show that $\dfrac{\sin x\tan x}{1-\cos x} = 1+\sec x$.

(4 marks)

7 (a) Given that $2\cos x = 3\tan x$, show that $2\sin^2 x + 3\sin x - 2 = 0$.

(3 marks)

(b) Hence solve $2\cos\left(2\alpha+\frac{\pi}{4}\right)=3\tan\left(2\alpha+\frac{\pi}{4}\right)$ for $0<\alpha<\pi$ radians, giving your answers in terms of π.

(4 marks)

8 Solve the equation $5\tan x-3\cot x = 2\sec x$ for $0° \leq x \leq 360°$.

(6 marks)

9 (a) Throughout this question, $A=\frac{\tan x}{1+\sec x}$ and $B=\frac{\tan x}{1-\sec x}$.

Show that $AB=k$ where k is a constant to be found.

(2 marks)

(b) Show that $A+B=-2\cot x$.

(4 marks)

(c) Use your answers to part (a) and part (b) to simplify the expression $(A+1)(B+1)$.

Hence, or otherwise, solve

$$\left(\frac{\tan(x-60)}{1+\sec(x-60)}+1\right)\left(\frac{\tan(x-60)}{1-\sec(x-60)}+1\right)=2\sqrt{3}$$

for $-180° < x < 180°$.

(5 marks)