IGCSE · Cambridge (CIE) · Further Maths   1 hour   12 questions

Exam Questions

Vectors in Two Dimensions

Introduction to Vectors / Vector Addition / Problem Solving using Vectors / Compose & Resolve Velocities

Medium (2 questions)	/14
Hard (5 questions)	/35
Very Hard (5 questions)	/38
Total Marks	**/87**

Medium Questions

1 (a) Find the unit vector in the direction of $\begin{pmatrix} 5 \\ -12 \end{pmatrix}$.

(1 mark)

(b) Given that $\begin{pmatrix} 4 \\ 1 \end{pmatrix} + k\begin{pmatrix} -2 \\ 3 \end{pmatrix} = r\begin{pmatrix} -10 \\ 5 \end{pmatrix}$, find the value of each of the constants k and r.

(3 marks)

(c) Relative to an origin O, the points A, B and C have position vectors $\mathbf{p}$, $3\mathbf{q} - \mathbf{p}$ and $9\mathbf{q} - 5\mathbf{p}$ respectively.

(i) Find $\overrightarrow{AB}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

[1]

(ii) Find $\overrightarrow{AC}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

[1]

(iii) Explain why A, B and C all lie in a straight line.

[1]

(iv) Find the ratio $AB : BC$.

[1]

(4 marks)

2 (a) The unit vectors $\mathbf{i}$ and $\mathbf{j}$ represent due east and due north respectively.

Person A starts at a position of $(-4\mathbf{i}+\mathbf{j})$ metres and walks with a constant velocity of $(3\mathbf{i}-\mathbf{j})$ metres per second.

Find the position vector of person A after 5 seconds.

(1 mark)

(b) Person B walks with a constant velocity of $(2\mathbf{i}+2\sqrt{3}\mathbf{j})$ metres per second.

Find

(i) the speed of person B,

(ii) the bearing of the direction in which person B is walking.

(5 marks)

Hard Questions

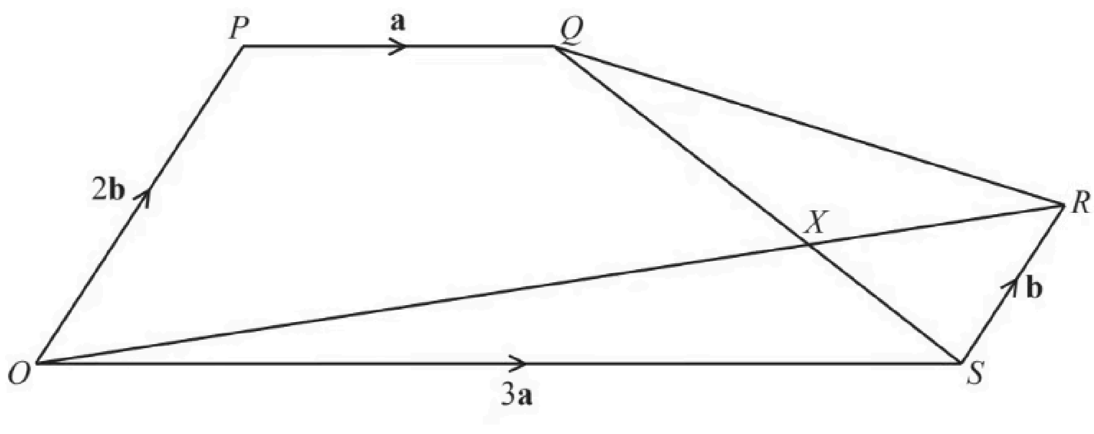
1 The parallelogram $OABC$ is such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. The point D lies on OC such that $OD : DC = 1 : 2$. The point E lies on AC such that $AE : EC = 2 : 1$.

Show that $\overrightarrow{OB} = k\overrightarrow{DE}$, where k is an integer to be found.

(5 marks)

2 (a) In the diagram $\overrightarrow{OP} = 2\mathbf{b}$, $\overrightarrow{OS} = 3\mathbf{a}$, $\overrightarrow{SR} = \mathbf{b}$ and $\overrightarrow{PQ} = \mathbf{a}$. The lines OR and QS intersect at X.

Find $\overrightarrow{OQ}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.



(1 mark)

(b) Find $\overrightarrow{QS}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(1 mark)

(c) Given that $\overrightarrow{QX} = \mu\overrightarrow{QS}$, find $\overrightarrow{OX}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and μ.

(1 mark)

(d) Given that $\overrightarrow{OX} = \lambda\overrightarrow{OR}$, find $\overrightarrow{OX}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and λ.

(1 mark)

(e) Find the value of λ and of μ.

(3 marks)

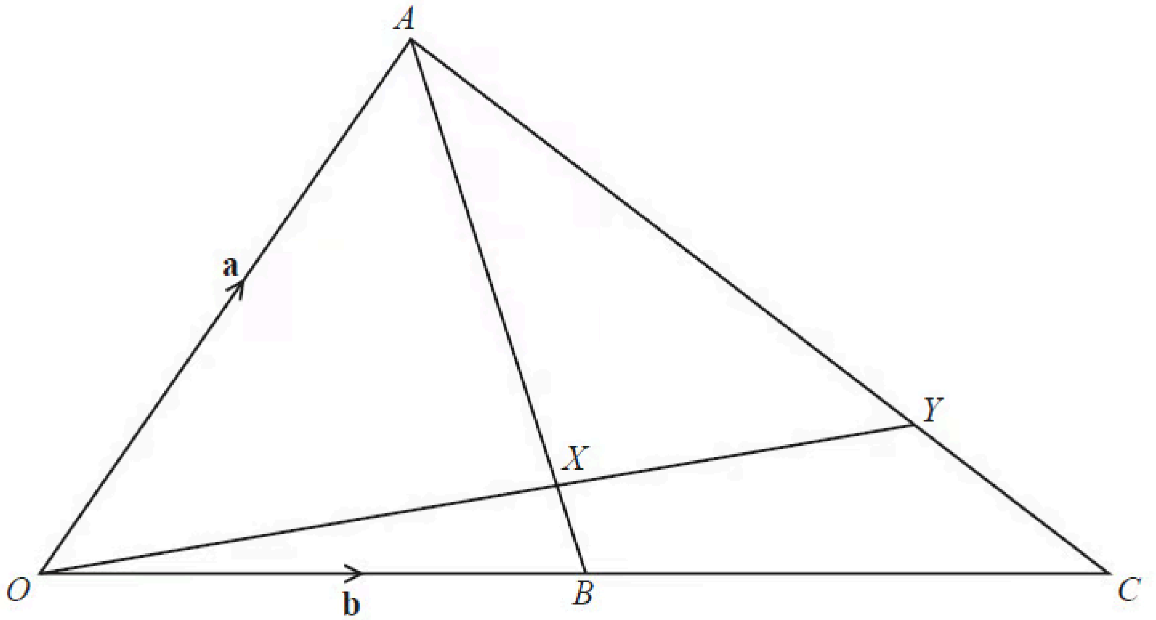
(f) Find the value of $\dfrac{QX}{XS}$.

(1 mark)

(g) Find the value of $\dfrac{OR}{OX}$.

(1 mark)

3 (a)



The diagram shows the triangle OAC. The point B is the midpoint of OC. The point Y lies on AC such that OY intersects AB at the point X where $AX : XB = 3:1$. It is given that $\overrightarrow{OA} = a$ and $\overrightarrow{OB} = b$.

Find $\overrightarrow{OX}$ in terms of a and b, giving your answer in its simplest form.

(3 marks)

(b) Find *AC* in terms of **a** and **b**.

(1 mark)

(c) Given that $\overrightarrow{OY} = h\overrightarrow{OX}$, find $\overrightarrow{AY}$ in terms of a, b and h.

(1 mark)

(d) Given that $\overrightarrow{AY} = m\overrightarrow{AC}$, find the value of h and of m.

(4 marks)

4 (a) The vectors $\mathbf{a}$ and $\mathbf{b}$ are such that $\mathbf{a} = \alpha\mathbf{i} + \mathbf{j}$ and $\mathbf{b} = 12\mathbf{i} + \beta\mathbf{j}$.

Find the value of each of the constants α and β such that $4\mathbf{a} - \mathbf{b} = (\alpha + 3)\mathbf{i} - 2\mathbf{j}$.

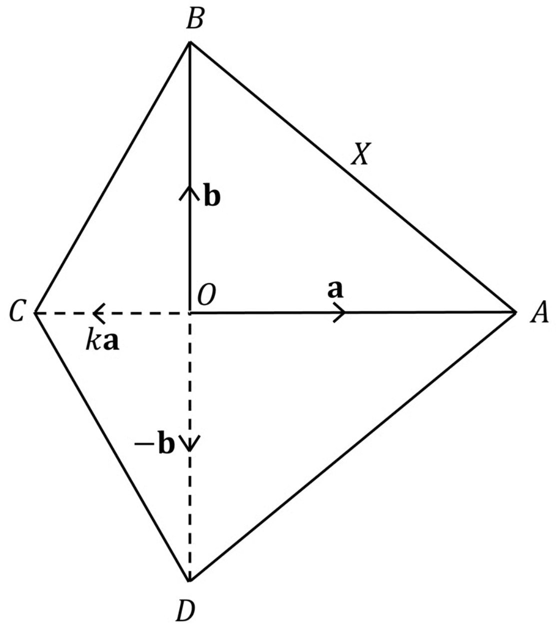
(3 marks)

(b) Hence find the unit vector in the direction of $\mathbf{b} - 4\mathbf{a}$.

(2 marks)

5 (a) The kite $ABCD$ has diagonals CA and BD which intersect at O, where $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{OC} = k\mathbf{a}$ and $\overrightarrow{OD} = -\mathbf{b}$.

The point X is the midpoint of AB, as shown.



Find

(i) $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$

(ii) $\overrightarrow{OX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$

(iii) $\overrightarrow{CD}$ in terms of k, $\mathbf{a}$ and $\mathbf{b}$

(3 marks)

(b) The point Y lies on CD such that $CY : YD = 1 : 2$.

If $\overrightarrow{YX}$ is parallel to $\mathbf{a} + \mathbf{b}$, find k.

(4 marks)

Very Hard Questions

1 (a) A particle P is initially at the point with position vector $\begin{pmatrix} 30 \\ 10 \end{pmatrix}$ and moves with a constant speed of $10\ \mathrm{ms}^{-1}$ in the same direction as $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$.

Find the position vector of P after t s.

(3 marks)

(b) As P starts moving, a particle Q starts to move such that its position vector after t s is given by $\begin{pmatrix} -80 \\ 90 \end{pmatrix} + t\begin{pmatrix} 5 \\ 12 \end{pmatrix}$.

Write down the speed of Q.

(1 mark)

(c) Find the exact distance between P and Q when $t = 10$, giving your answer in its simplest surd form.

(3 marks)

2 (a) Relative to an origin O, the position vectors of the points A, B, C and D are

$$\overrightarrow{OA} = \begin{pmatrix} 6 \\ -5 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 10 \\ 3 \end{pmatrix}, \overrightarrow{OC} = \begin{pmatrix} x \\ y \end{pmatrix} \text{ and } \overrightarrow{OD} = \begin{pmatrix} 12 \\ 7 \end{pmatrix}$$

Find the unit vector in the direction of $\overrightarrow{AB}$.

(3 marks)

(b) The point A is the mid-point of BC. Find the value of x and of y.

(2 marks)

(c) The point E lies on OD such that $OE : OD$ is $1 : 1 + \lambda$. Find the value of λ such that $\overrightarrow{BE}$ is parallel to the x-axis.

(3 marks)

3 (a) In this question all distances are in km.

A ship P sails from a point A, which has position vector $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, with a speed of $52\ \mathrm{kmh^{-1}}$ in the direction of $\begin{pmatrix} -5 \\ 12 \end{pmatrix}$

Find the velocity vector of the ship.

(1 mark)

(b) Write down the position vector of P at a time t hours after leaving A.

(1 mark)

(c) At the same time that ship P sails from A, a ship Q sails from a point B, which has position vector $\begin{pmatrix} 12 \\ 8 \end{pmatrix}$, with velocity vector $\begin{pmatrix} -25 \\ 45 \end{pmatrix}\mathrm{kmh^{-1}}$.

Write down the position vector of Q at a time t hours after leaving B.

(1 mark)

(d) Using your answers to parts (b) and (c), find the displacement vector $\overrightarrow{PQ}$ at time t hours.

(1 mark)

(e) Hence show that $PQ = \sqrt{34t^2 - 168t + 208}$.

(2 marks)

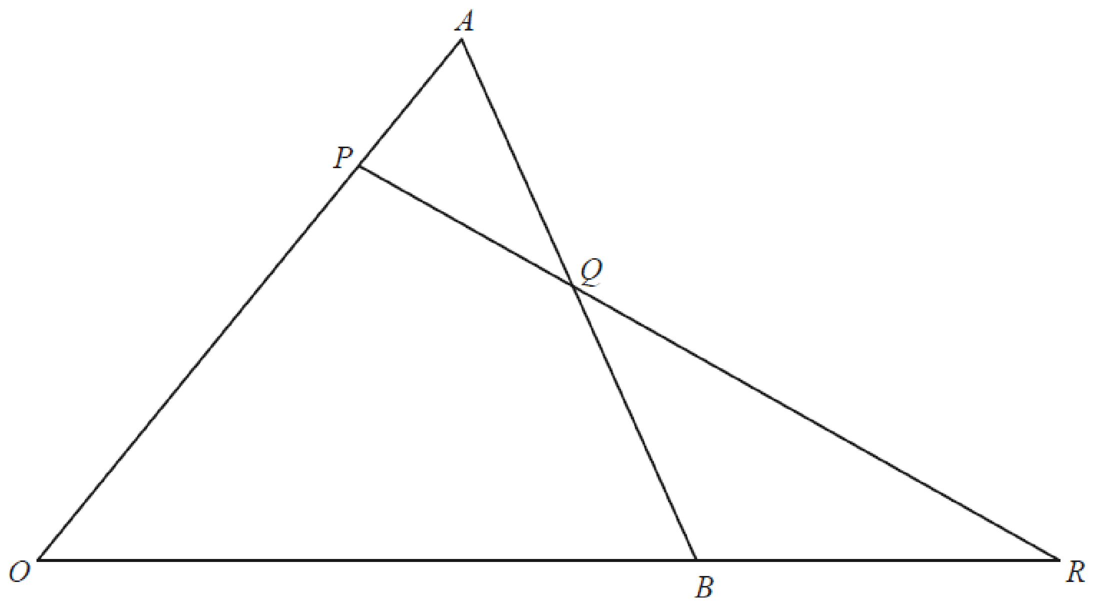
(f) Find the value of t when P and Q are first $2\ \mathrm{km}$ apart.

(2 marks)

4 The position vectors of three points, A, B and C, relative to an origin O, are $\begin{pmatrix} -5 \\ -7 \end{pmatrix}$, $\begin{pmatrix} 10 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} x \\ y \end{pmatrix}$ respectively. Given that $\overrightarrow{AC} = 4\overrightarrow{BC}$, find the unit vector in the direction of $\overrightarrow{OC}$.

(5 marks)

5 (a)



The diagram shows a triangle OAB such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point P lies on OA such that $OP = \frac{3}{4}OA$. The point Q is the mid-point of AB. The lines OB and PQ are extended to meet at the point R. Find, in terms of $\mathbf{a}$ and $\mathbf{b}$,

Find $\overrightarrow{AB}$

(1 mark)

(b) Find $\overrightarrow{PQ}$. Give your answer in its simplest form.

(3 marks)

(c) It is given that $n\overrightarrow{PQ} = \overrightarrow{QR}$ and $\overrightarrow{BR} = k\mathbf{b}$, where n and k are positive constants.

Find $\overrightarrow{QR}$ in terms of n, $\mathbf{a}$ and $\mathbf{b}$.

(1 mark)

(d) Find $\overrightarrow{QR}$ in terms of k, $\mathbf{a}$ and $\mathbf{b}$.

(2 marks)

(e) Hence find the value of n and of k.

(3 marks)